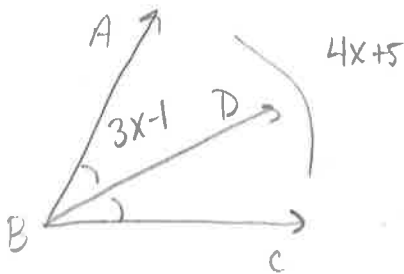


1.6 - Angle Pairs

Bellwork

\overrightarrow{BD} bisects $\angle ABC$, $m\angle ABC = 4x + 5$, and $m\angle ABD = 3x - 1$. What is the value of x ?



$$\begin{aligned}
 2(3x-1) &= 4x+5 \\
 6x-2 &= 4x+5 \\
 -4x &\quad -4x \\
 \hline
 2x-2 &= 5 \\
 +2 &\quad +2 \\
 \hline
 2x &= 7 \\
 \frac{2x}{2} &\quad \frac{7}{2} \\
 x &= \frac{7}{2} \text{ or } 3.5
 \end{aligned}$$

Write the sentence as an equation and solve.

1. The difference between a number and 14 is 8.

$$\begin{array}{r}
 x - 14 = 8 \\
 +14 \quad +14 \\
 \hline
 x = 22
 \end{array}$$

2. Twice the difference between 5 times a number and 6 is 18.

$$\begin{array}{r}
 2(5x-6) = 18 \\
 10x-12 = 18 \\
 \hline
 10x = 30 \\
 \frac{10x}{10} = \frac{30}{10} \\
 x = 3
 \end{array}$$

3. Fourteen is 7 times the difference between a number and 2.

$$\begin{array}{r}
 \frac{14}{7} = \frac{7(x-2)}{7} \\
 2 = x-2 \\
 +2 \quad +2 \\
 \hline
 4 = x
 \end{array}$$

4. Four consecutive odd integers such that 2 times the last integer is 5 more than the sum of the first 3 integers.

$$x, x+2, x+4, x+6$$

$$\begin{aligned}
 2(x+6) &= x+(x+2)+(x+4)+5 \\
 2x+12 &= 3x+11 \\
 -2x \quad -11 &\quad -2x \quad -11 \\
 \hline
 1 &= x
 \end{aligned}$$

$$1 = x$$

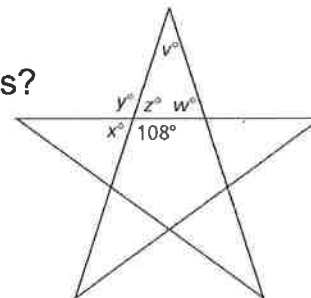
Work with a partner. The five-pointed star has a regular pentagon at its center.

a. What do you notice about the following angle pairs?

x° and y°

y° and z°

x° and z°



b. Find the values of the indicated variables. Do not use a protractor to measure the angles.

$x =$

$y =$

$z =$

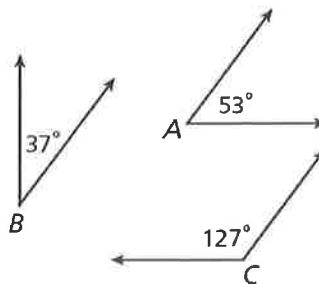
$w =$

$v =$

Explain how you obtained each answer.

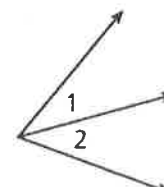
Complementary and Supplementary Angles

Complementary angles are two angles whose measures have a sum of 90° .
 $\angle A$ and $\angle B$ are complementary.



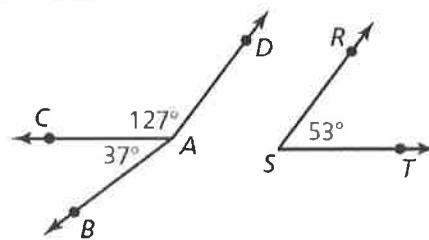
Supplementary angles are two angles whose measures have a sum of 180° .
 $\angle A$ and $\angle C$ are supplementary.

Adjacent angles are two angles in the same plane with a common vertex and a common side, but no common interior points. $\angle 1$ and $\angle 2$ are adjacent angles.



(Angles that are side by side; they share a side, but one of them cannot be inside the other)

In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.



$\angle BAC$ and $\angle RST$ are complementary
 $\angle CAD$ and $\angle RST$ are supplementary
 $\angle BAC$ and $\angle CAD$ are adjacent

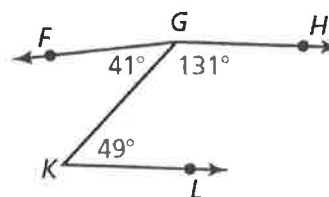
a. $\angle 1$ is a complement of $\angle 2$, and $m\angle 1 = 62^\circ$. Find $m\angle 2$.

$$\begin{array}{r} m\angle 1 + m\angle 2 = 90 \\ \angle 2 + m\angle 2 = 90 \\ -\angle 2 \quad -\angle 2 \\ \hline m\angle 2 = 28^\circ \end{array}$$

b. $\angle 3$ is a supplement of $\angle 4$, and $m\angle 4 = 47^\circ$. Find $m\angle 3$.

$$\begin{array}{r} m\angle 3 + m\angle 4 = 180 \\ m\angle 3 + 47 = 180 \\ -47 \quad -47 \\ \hline m\angle 3 = 133^\circ \end{array}$$

In Exercises 1 and 2, use the figure.



1. Name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.

$\angle FGK$ and $\angle LKG$ are complementary
 $\angle HGK$ and $\angle LKG$ are supplementary

$\angle FGK$ and $\angle HGK$ are adjacent

2. Are $\angle KGH$ and $\angle LKG$ adjacent angles? Are $\angle FGK$ and $\angle FGH$ adjacent angles? Explain.

No, they share interior points

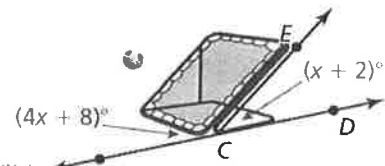
3. $\angle 1$ is a complement of $\angle 2$, and $m\angle 2 = 5^\circ$. Find $m\angle 1$.

$$\begin{array}{r} m\angle 1 + m\angle 2 = 90 \\ m\angle 1 + 5 = 90 \\ -5 \quad -5 \\ \hline m\angle 1 = 85^\circ \end{array}$$

4. $\angle 3$ is a supplement of $\angle 4$, and $m\angle 3 = 148^\circ$. Find $m\angle 4$.

$$\begin{array}{r} m\angle 3 + m\angle 4 = 180 \\ 148 + m\angle 4 = 180 \\ -148 \quad -148 \\ \hline m\angle 4 = 32^\circ \end{array}$$

When viewed from the side, the frame of a ball-return net forms a pair of supplementary angles with the ground. Find $m\angle BCE$ and $m\angle ECD$.



$\angle BCE$ and $\angle DCE$ are supplementary
 $m\angle BCE + m\angle DCE = 180^\circ$
 - Definition of Supplementary

$$4x + 8 + x + 2 = 180$$

$$5x + 10 = 180$$

$$\begin{array}{r} 5x + 10 = 180 \\ -10 \quad -10 \\ \hline 5x = 170 \\ \hline x = 34 \end{array}$$

$$m\angle BCE = 4(34) + 8 = 136 + 8$$

$$m\angle BCE = 144^\circ$$

$$m\angle ECD = 34 + 2 = 36^\circ$$

$\angle LMN$ and $\angle PQR$ are complementary angles. Find the measures of the angles when $m\angle LMN = (4x - 2)^\circ$ and $m\angle PQR = (9x + 1)^\circ$.

$$4x - 2 + 9x + 1 = 90$$

$$13x - 1 = 90$$

$$\begin{array}{r} 13x - 1 = 90 \\ +1 \quad +1 \\ \hline 13x = 91 \\ \hline x = 7 \end{array}$$

$$x = 7$$

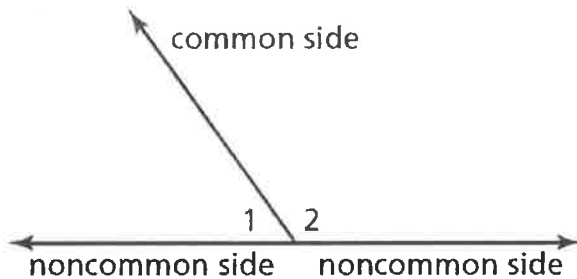
$$m\angle LMN = 4(7) - 2 = 26^\circ$$

$$m\angle PQR = 9(7) + 1 = 64^\circ$$

Core Concept

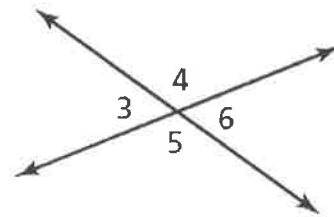
Linear Pairs and Vertical Angles

Two adjacent angles are a **linear pair** when their noncommon sides are opposite rays. The angles in a linear pair are supplementary angles.



$\angle 1$ and $\angle 2$ are a linear pair.

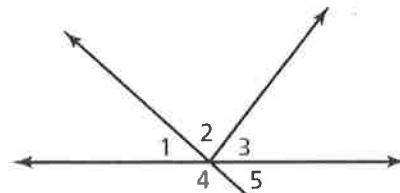
Two angles are **vertical angles** when their sides form two pairs of opposite rays.



$\angle 3$ and $\angle 6$ are vertical angles.

$\angle 4$ and $\angle 5$ are vertical angles.

Identify all the linear pairs and all the vertical angles in the figure.



Linear Pairs

$\angle 4$ and $\angle 5$
 $\angle 1$ and $\angle 4$

Vertical \angle 's

$\angle 1$ and $\angle 5$

Adjacent Only (Not Linear Pairs)

$\angle 1$ and $\angle 2$
 $\angle 2$ and $\angle 3$
 $\angle 3$ and $\angle 5$

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

$\angle 5$ and $\angle 6$

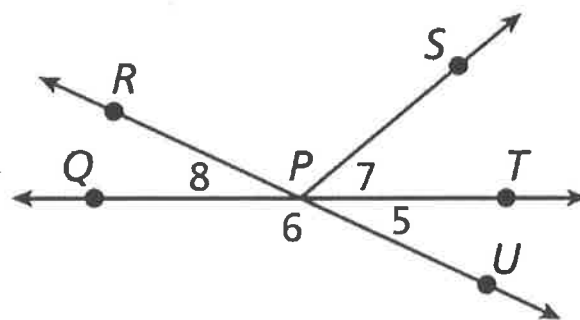
Adjacent and Linear Pair

$\angle 7$ and $\angle SPU$

Not Adjacent

$\angle 7$ and $\angle 8$

Not Adjacent



Two angles form a linear pair. The measure of one angle is five times the measure of the other angle. Find the measure of each angle.

$$X + 5X = 180$$

$$\frac{6X}{6} = \frac{180}{6}$$

$$X = 30$$

$X = 30$
 $5X = 150$

The measure of an angle is twice the measure of its complement. Find the measure of each angle.

$$X + 2X = 90$$

$$\frac{3X}{3} = \frac{90}{3}$$

$$X = 30$$

$X = 30$
 $2X = 60$

Two angles form a linear pair. The measure of one angle is 1.5 times the measure of the other angle. Find the measure of each angle.

$$X + 1.5X = 180$$

$$\frac{2.5X}{2.5} = \frac{180}{2.5}$$

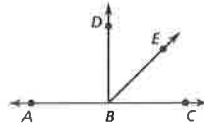
$$X = 72$$

$X = 72$
 $1.5X = 108$

Concept Summary

Interpreting a Diagram

There are some things you can conclude from a diagram, and some you cannot. For example, here are some things that you *can* conclude from the diagram below.



YOU CAN CONCLUDE

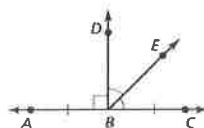
- All points shown are coplanar.
- Points A, B, and C are collinear, and B is between A and C.
- \overline{AC} , \overline{BD} , and \overline{BE} intersect at point B.
- $\angle DBE$ and $\angle EBC$ are adjacent angles, and $\angle ABC$ is a straight angle.
- Point E lies in the interior of $\angle DBC$.

Here are some things you *cannot* conclude from the diagram above.

YOU CANNOT CONCLUDE

- $\overline{AB} \cong \overline{BC}$.
- $\angle DBE \cong \angle EBC$.
- $\angle ABD$ is a right angle.

To make such conclusions, the following information must be given.



Homework

pg. 52 # 3-6, 11-22, 32 - 41