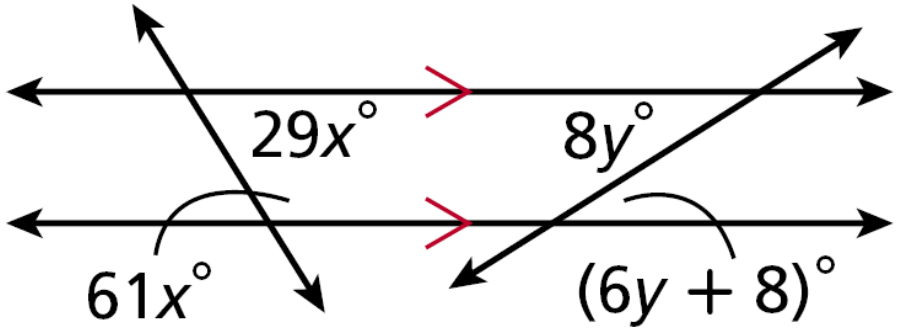
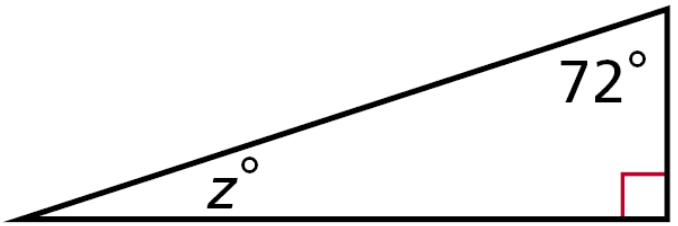
Warm Up

Find the value of each variable.





1. x 2

2. y 4

3. z 18

6-2 Properties of Parallelograms

Objectives

Prove and apply properties of parallelograms.

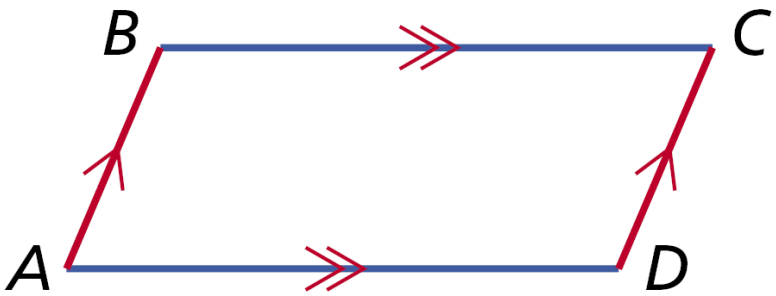
Use properties of parallelograms to solve problems.

6-2 Properties of Parallelograms

Parallelogram a quadrilateral with two pairs of parallel sides

Parallelogram $ABCD$

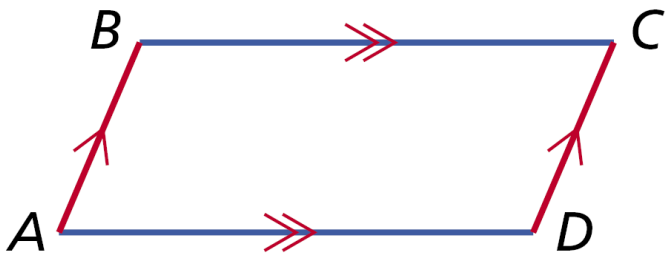
$\square ABCD$



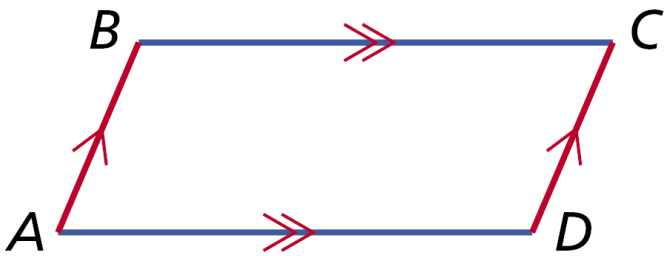
$\overline{AB} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{DA}$

Properties of Parallelograms:

- **Opposite sides are congruent**

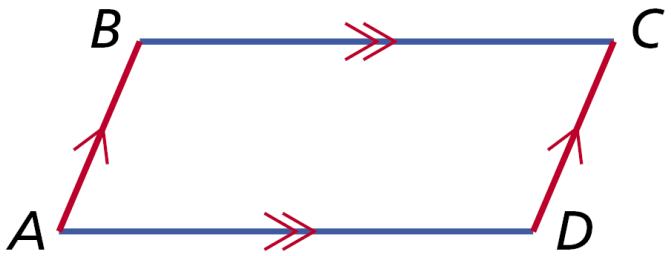


- **Opposite angles are congruent**

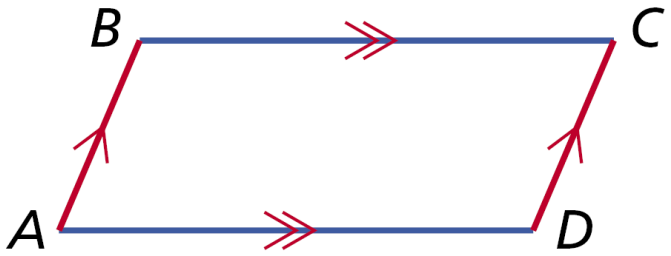


6-2 Properties of Parallelograms

- **Consecutive angles are supplementary**

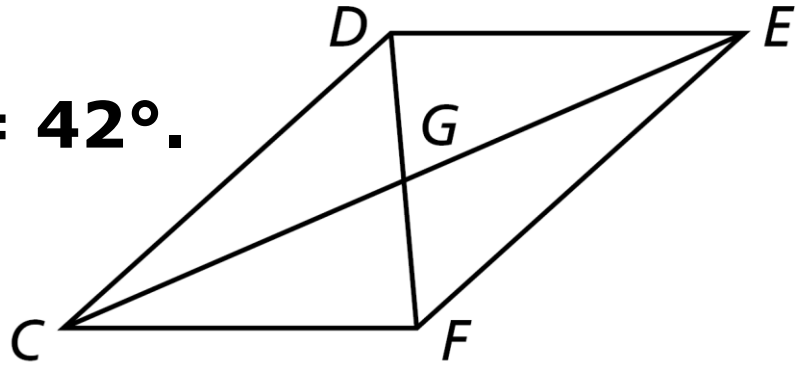


- **The Diagonals bisect each other.**



Example 1A: Properties of Parallelograms

In $\square CDEF$, $DE = 74$ mm, $DG = 31$ mm, and $m\angle FCD = 42°$. Find CF.



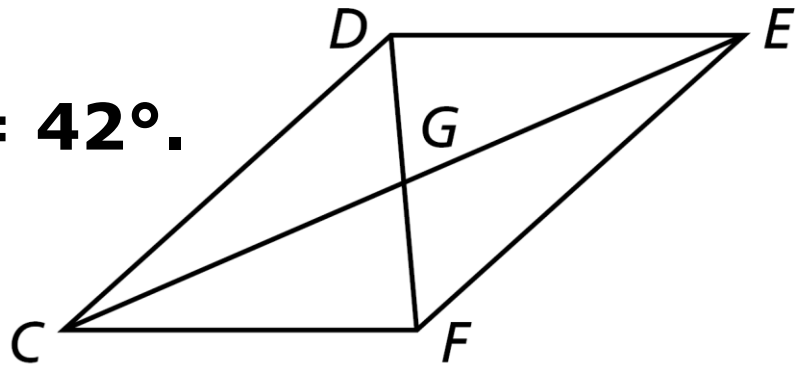
$\overline{CF} \cong \overline{DE}$ — $\square$ → *opp. sides* $\cong$

$CF = DE$ — *Def. of* $\cong$ *segs.*

$CF = 74$ mm — *Substitute 74 for DE.*

Example 1B: Properties of Parallelograms

In $\square CDEF$, $DE = 74$ mm, $DG = 31$ mm, and $m\angle FCD = 42°$. Find $m\angle EFC$.



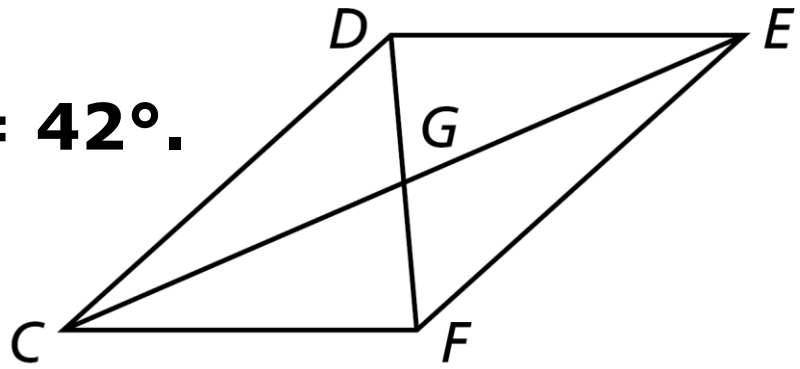
$m\angle EFC + m\angle FCD = 180°$ $\square \rightarrow$ *cons. ∠s supp.*

$m\angle EFC + 42 = 180$ *Substitute 42 for $m\angle FCD$.*

$m\angle EFC = 138°$ *Subtract 42 from both sides.*

Example 1C: Properties of Parallelograms

In $\square CDEF$, $DE = 74$ mm, $DG = 31$ mm, and $m\angle FCD = 42°$. Find DF.



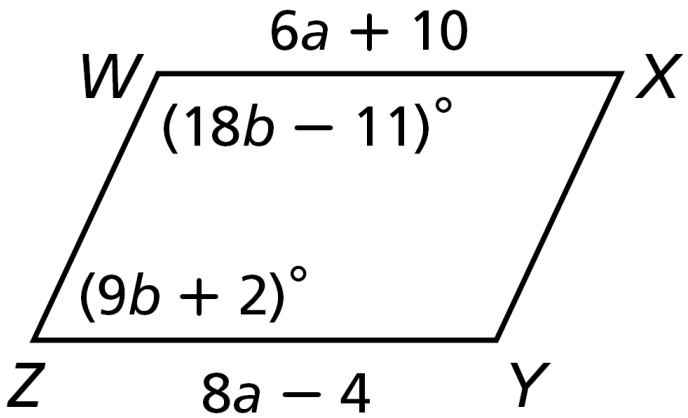
$DF = 2DG$ $\square$ → *diags. bisect each other.*

$DF = 2(31)$ *Substitute 31 for DG.*

$DF = 62$ *Simplify.*

Example 2A: Using Properties of Parallelograms to Find Measures

***WXYZ* is a parallelogram. Find *YZ*.**



$\overline{YZ} \cong \overline{XW}$ — □ → opp. ∠s ≅

$YZ = XW$ — Def. of ≅ segs.

$8a - 4 = 6a + 10$ — Substitute the given values.

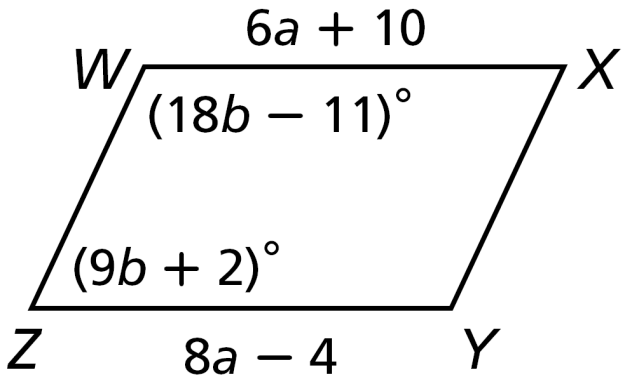
$2a = 14$ — Subtract 6a from both sides and add 4 to both sides.

$a = 7$ — Divide both sides by 2.

$$YZ = 8a - 4 = 8(7) - 4 = 52$$

Example 2B: Using Properties of Parallelograms to Find Measures

***WXYZ* is a parallelogram. Find m∠*Z* .**



$m\angle Z + m\angle W = 180°$ ▱ → *cons. ∠s supp.*

$(9b + 2) + (18b - 11) = 180$ *Substitute the given values.*

$27b - 9 = 180$ *Combine like terms.*

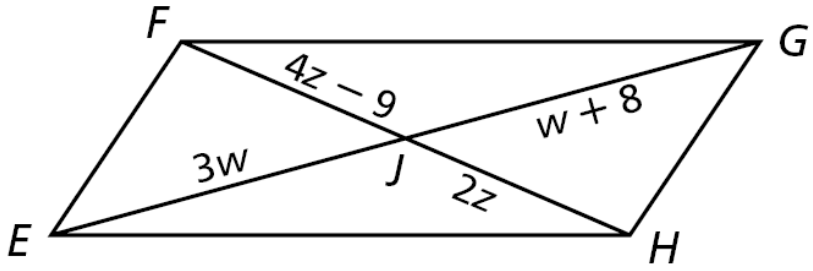
$27b = 189$ *Add 9 to both sides.*

$b = 7$ *Divide by 27.*

$m\angle Z = (9b + 2)° = [9(7) + 2]° = 65°$

Check It Out! Example 2a

***EFGH* is a parallelogram. Find *JG*.**



$\overline{EJ} \cong \overline{JG}$ — $\square$ → *diags. bisect each other.*

$EJ = JG$ — *Def. of ≅ segs.*

$3w = w + 8$ — *Substitute.*

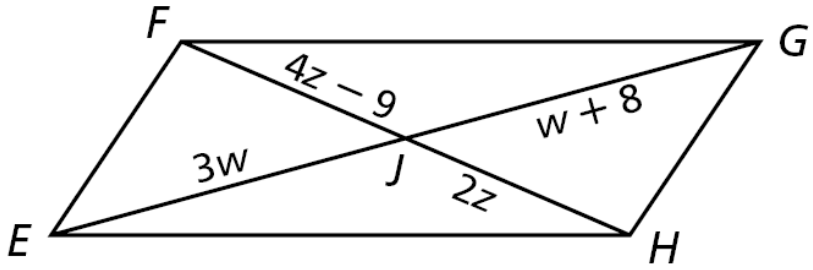
$2w = 8$ — *Simplify.*

$w = 4$ — *Divide both sides by 2.*

$JG = w + 8 = 4 + 8 = 12$

Check It Out! Example 2b

***EFGH* is a parallelogram. Find *FH*.**



$\overline{FJ} \cong \overline{JH}$ — $\square$ → *diags. bisect each other.*

$FJ = JH$ — *Def. of ≅ segs.*

$4z - 9 = 2z$ — *Substitute.*

$2z = 9$ — *Simplify.*

$z = 4.5$ — *Divide both sides by 2.*

$FH = (4z - 9) + (2z) = 4(4.5) - 9 + 2(4.5) = 18$

Remember!

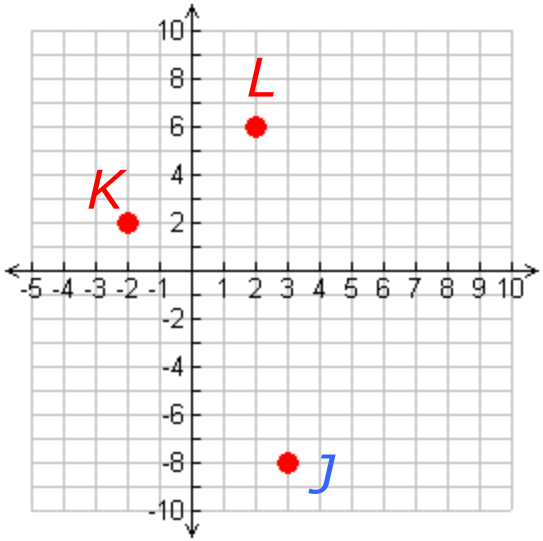
When you are drawing a figure in the coordinate plane, the name *ABCD* gives the order of the vertices.

Example 3: Parallelograms in the Coordinate Plane

Three vertices of $\square JKLM$ are $J(3, -8)$, $K(-2, 2)$, and $L(2, 6)$. Find the coordinates of vertex M.

Since *JKLM* is a parallelogram, both pairs of opposite sides must be parallel.

Step 1 Graph the given points.

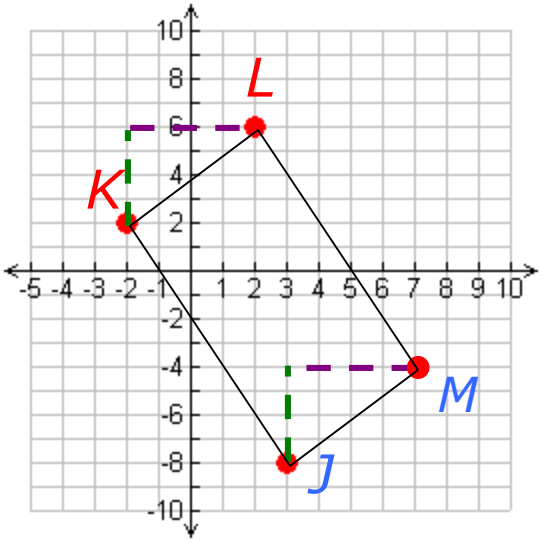


Example 3 Continued

Step 2 Find the slope of $\overline{KL}$ by counting the units from *K* to *L*.

The rise from 2 to 6 is 4.

The run of –2 to 2 is 4.



Step 3 Start at *J* and count the same number of units.

A rise of 4 from –8 is –4.

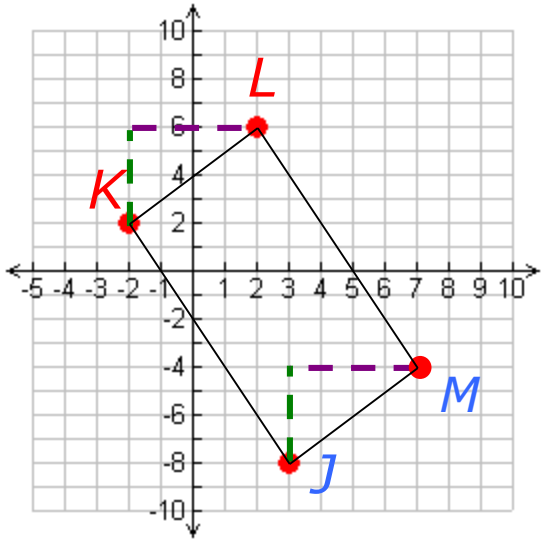
A run of 4 from 3 is 7. Label (7, –4) as vertex *M*.

Example 3 Continued

Step 4 Use the slope formula to verify that $\overline{LM} \parallel \overline{KJ}$.

$$\text{slope of } \overline{LM} = \frac{-4-6}{7-2} = \frac{-10}{5} = -2$$

$$\text{slope of } \overline{KJ} = \frac{-8-2}{3-(-2)} = \frac{-10}{5} = -2$$



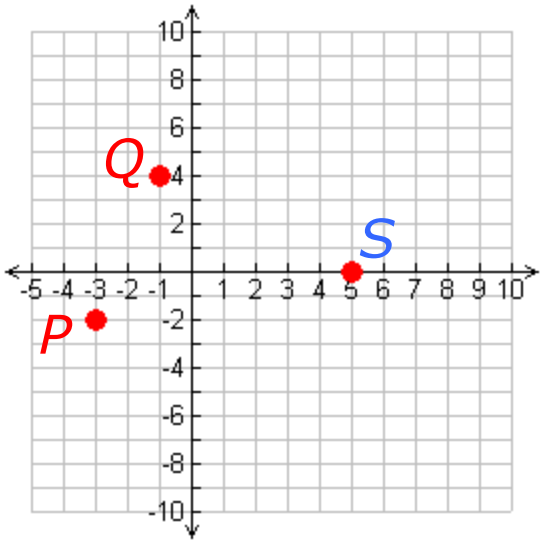
The coordinates of vertex *M* are (7, –4).

Check It Out! Example 3

Three vertices of $\square PQRS$ are $P(-3, -2)$, $Q(-1, 4)$, and $S(5, 0)$. Find the coordinates of vertex R.

Since *PQRS* is a parallelogram, both pairs of opposite sides must be parallel.

Step 1 Graph the given points.

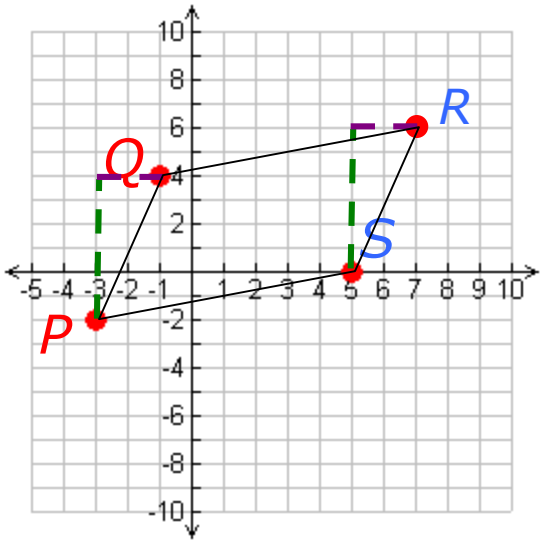


Check It Out! Example 3 Continued

Step 2 Find the slope of $\overline{PQ}$ by counting the units from P to Q.

The rise from -2 to 4 is 6.

The run of -3 to -1 is 2.



Step 3 Start at S and count the same number of units.

A rise of 6 from 0 is 6.

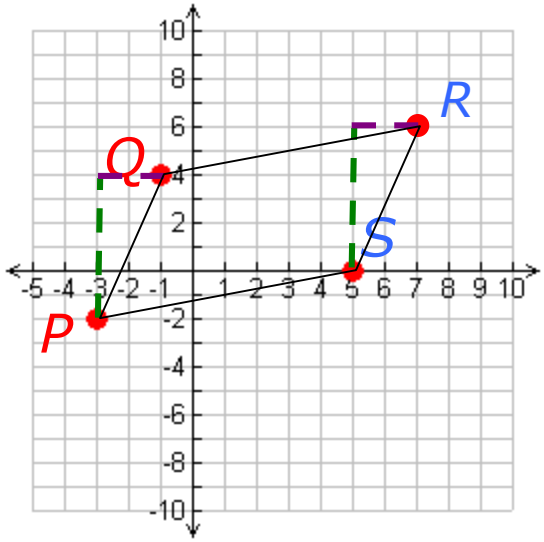
A run of 2 from 5 is 7. Label $(7, 6)$ as vertex R.

Check It Out! Example 3 Continued

Step 4 Use the slope formula to verify that $\overline{PQ} \parallel \overline{SR}$.

$$\text{slope of } \overline{PQ} = \frac{-2-4}{-3-(-1)} = \frac{-6}{-2} = 3$$

$$\text{slope of } \overline{SR} = \frac{0-6}{5-7} = \frac{-6}{-2} = 3$$



The coordinates of vertex *R* are (7, 6).