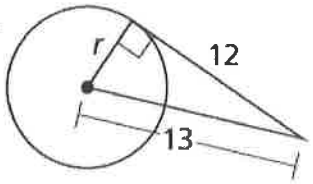
10.1 Lines that Intersect Circles

Bellwork:

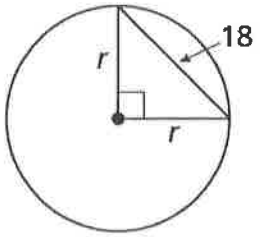
Find the radius of the circle.

1.



$r = 5$

2.



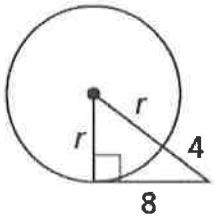
$l\sqrt{2} = 18$

$l = \frac{18}{\sqrt{2}} = \frac{18\sqrt{2}}{2}$

$l = 9\sqrt{2}$

$r = 9\sqrt{2}$

3.



$r^2 + 8^2 = (r+4)^2$

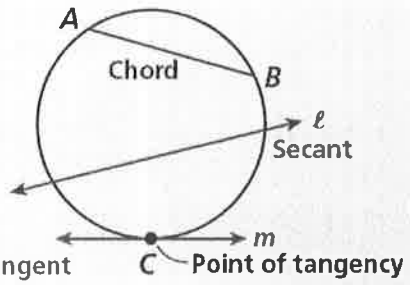
$r^2 + 64 = r^2 + 8r + 16$

$\frac{48}{8} = \frac{8r}{8}$

$6 = r$

a **circle** is the set of all points in a plane that are equidistant from a given point, called the center of the circle. A circle with center *C* is called circle *C*, or ⊙*C*.

Lines and Segments That Intersect Circles

TERM	DIAGRAM
A **chord** is a segment whose endpoints lie on a circle.	
A **secant** is a line that intersects a circle at two points.	
A **tangent** is a line in the same plane as a circle that intersects it at exactly one point.	
The point where the tangent and a circle intersect is called the **point of tangency**.	

** Radius: endpoints are on the center and any point ON the circle

** Diameter: chord that contains the center of a circle. (Note: it is twice the radius)

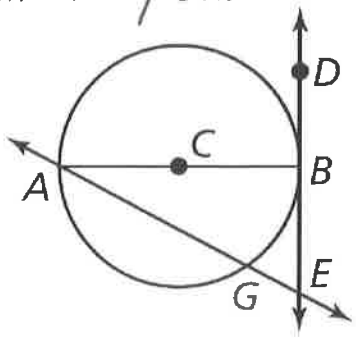
Tell whether the line, ray, or segment is best described as a *radius*, *chord*, *diameter*, *secant*, or *tangent* of $\odot C$.

a. $\overline{AC}$ radius

b. $\overline{AB}$ diameter / chord

c. $\overleftrightarrow{DE}$ tangent

d. $\overleftrightarrow{AE}$ secant



e. what word best describes *AG* ? *CB* ?

$\overline{AG}$ is a chord

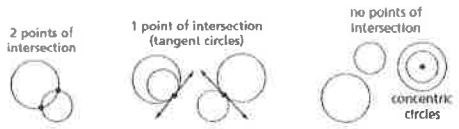
$\overline{CB}$ is a radius

f. name a tangent and a tangent segment.

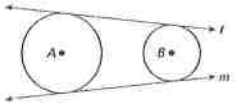
$\overleftrightarrow{DE}$ is a tangent

$\overline{DB}$ is a tangent segment

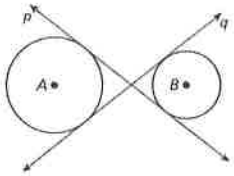
In a plane, two circles can intersect in two points, one point, or no points. Coplanar circles that intersect in one point are called **tangent circles**. Coplanar circles that have a common center are called **concentric circles**.



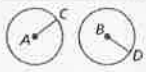
A **common tangent** is a line that is tangent to two circles.



Lines ℓ and m are common external tangents to $\odot A$ and $\odot B$.

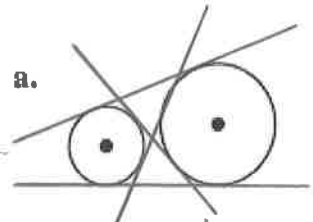


Lines p and q are common internal tangents to $\odot A$ and $\odot B$.

TERM	DIAGRAM
Two circles are **congruent circles** if and only if they have congruent radii.	 $\odot A \cong \odot B$ if $\overline{AC} \cong \overline{BD}$. $\overline{AC} \cong \overline{BD}$ if $\odot A \cong \odot B$.

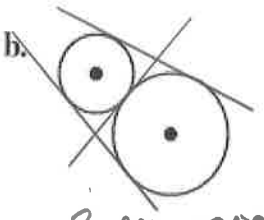
Tell how many common tangents the circles have and draw them. Use blue to indicate common external tangents and red to indicate common internal tangents.

a.



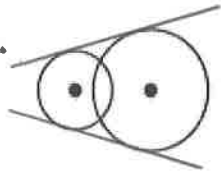
4 common tangents (2 internal, 2 external)

b.



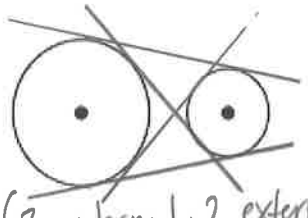
3 common tangents (2 external, 1 internal)

c.



2 common tangents (external)

3.



4 (2 internal, 2 external)

4.

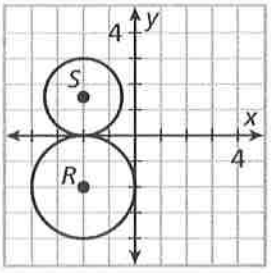
1 (external)

5.

None

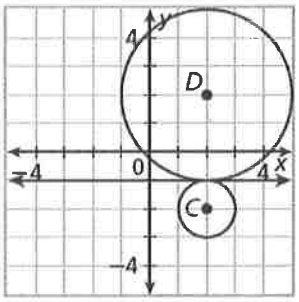
Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at this point.

radius of ⊙S = 1.5
radius of ⊙R = 2
point of tangency: (-2,0)
tangent line: y=0



Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at this point.

radius of ⊙D = 3
radius of ⊙C = 1
point of tangency = (2,-1)
tangent line: y=-1

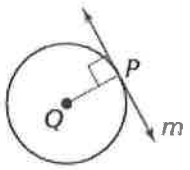


Homework for 10.1A:

pg. 534 #5-18

Theorem 10.1 Tangent Line to Circle Theorem

In a plane, a line is tangent to a circle if and only if the line is perpendicular to a radius of the circle at its endpoint on the circle.

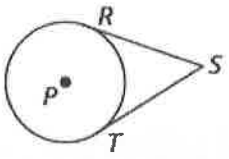


Line m is tangent to $\odot Q$ if and only if $m \perp \overline{QP}$.

Proof Ex. 47, p. 536

Theorem 10.2 External Tangent Congruence Theorem

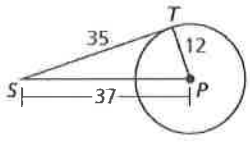
Tangent segments from a common external point are congruent.



If $\overline{SR}$ and $\overline{ST}$ are tangent segments, then $\overline{SR} \cong \overline{ST}$.

Proof Ex. 46, p. 536

Is $\overline{ST}$ tangent to $\odot P$?



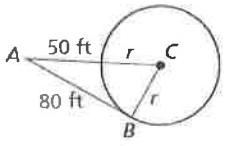
$12^2 + 35^2 \stackrel{?}{=} 37^2$

$144 + 1225 \stackrel{?}{=} 1369$

$1369 = 1369$ ✓

$\angle T$ is a right $\angle \rightarrow \overline{PT} \perp \overline{ST} \rightarrow$ ST is tangent

In the diagram, point B is a point of tangency. Find the radius r of $\odot C$.



$r^2 + 80^2 = (r + 50)^2$

$r^2 + 6400 = r^2 + 100r + 2500$

$3900 = 100r$

$\boxed{39 = r}$

Early in its flight, the Apollo 11 spacecraft orbited Earth at an altitude of 120 miles. What was the distance from the spacecraft to Earth's horizon rounded to the nearest mile?

(The radius of earth is ≈ 4000 miles)

How would you draw this picture?

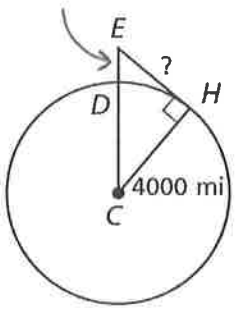
$4000^2 + (EH)^2 = 4120^2$

$16000000 + (EH)^2 = 16974400$

$(EH)^2 = 974400$

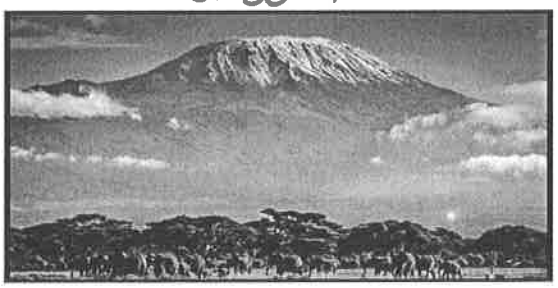
$EH = 987.1$ miles

ED = 120 miles



Kilimanjaro, the tallest mountain in Africa, is 19,340 ft tall. What is the distance from the summit of Kilimanjaro to the horizon to the nearest mile?

5280 ft = 1 mile

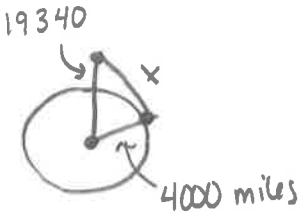


$\frac{19340 \text{ ft}}{1} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} = 3.66$ miles

$4000^2 + x^2 = 4003.66^2$

$x^2 = 29293.4$

$x = 171.2$ miles



If you climbed to the top of a 1768 foot tall guided tower, how far would it be to the horizon? (radius of earth ≈ 4000 mi.)

VIDEO: CLIMBING TO THE TOP OF A GUIDED TOWER TO CHANGE A LIGHTBULB

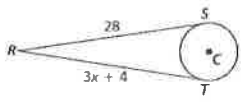
$\frac{1768\text{ ft}}{1} \cdot \frac{1\text{ mi}}{5280\text{ ft}} = .335\text{ miles}$

$4000^2 + x^2 = 4000.335^2$

$x^2 = 2678.9$

$x = 51.8$ miles

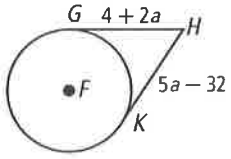
$\overline{RS}$ is tangent to $\odot C$ at S, and $\overline{RT}$ is tangent to $\odot C$ at T. Find the value of x.



$3x + 4 = 28$

$3x = 24$

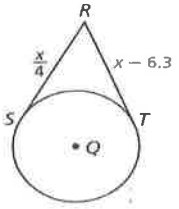
$x = 8$



$4 + 2a = 5a - 32$

$36 = 3a$

$12 = a$



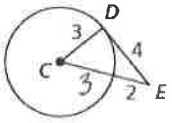
$\frac{x}{4} = x - 6.3$

$x = 4x - 25.2$

$-3x = -25.2$

$x = 8.4$

6. Is $\overline{DE}$ tangent to $\odot C$?

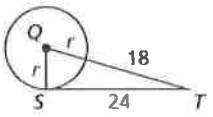


Yes - 3-4-5 $\triangle$

$\overline{DE} \perp \overline{DC}$

7. $\overline{ST}$ is tangent to $\odot Q$.

Find the radius of $\odot Q$.



$r^2 + 24^2 = (r+18)^2$

$r^2 + 576 = r^2 + 36r + 324$

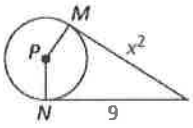
$576 = 36r + 324$

$252 = 36r$

$7 = r$

8. Points M and N are points of tangency.

Find the value(s) of x.



$x^2 = 9$

$x = \pm 3$

Homework 10.1B

pg. 534 #19-26, 29-32

Homework (Assign based on material covered):

pg. 534

#5-10 (Naming Segments)

#11-14 (Common Tangents)

#15-18 (Internal and External Tangents)

#19-22 (Determine if it's Tangent)

#23-26 (Find the radius given tangent and distance to the circle)

#29-32 (Tangents to the same external point)