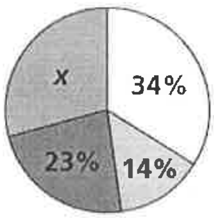
10.2 Finding Arc Measures

Bellwork:

Determine the value of *x* for the circle graph. Pay close attention to the units.

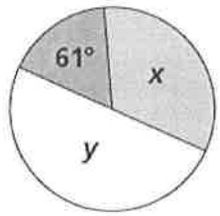
1.



$X = 100 - (34 + 14 + 23)$

$\boxed{X = 29}$

2.

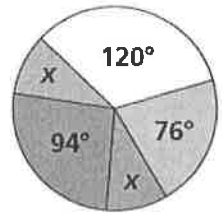


$y = 180°$

$X = 180 - 61$

$\boxed{X = 119°}$

3.



$360 - (120 + 94 + 76)$

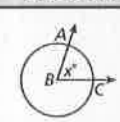
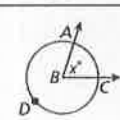
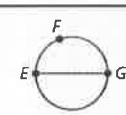
$360 - 290$

70

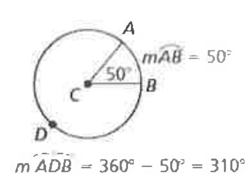
$\boxed{X = \frac{70}{2} = 35°}$

A **central angle** is an angle whose vertex is the center of a circle. An **arc** is a portion of a circle.

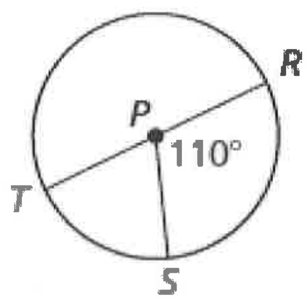
Arcs and Their Measure

ARC	MEASURE	DIAGRAM
A **minor arc** is an arc whose points are on or in the interior of a central angle.	The measure of a minor arc is equal to the measure of its central angle. $m\widehat{AC} = m\angle ABC = x°$	
A **major arc** is an arc whose points are on or in the exterior of a central angle.	The measure of a major arc is equal to 360° minus the measure of its central angle. $m\widehat{ADC} = 360° - m\angle ABC = 360° - x°$	
If the endpoints of an arc lie on a diameter, the arc is a **semicircle**.	The measure of a semicircle is equal to 180°. $m\widehat{EFG} = 180°$	

Minor arcs may be named by two points. Major arcs and semicircles must be named by three points.



Find the measure of each arc of $\odot P$, where $\overline{RT}$ is a diameter.



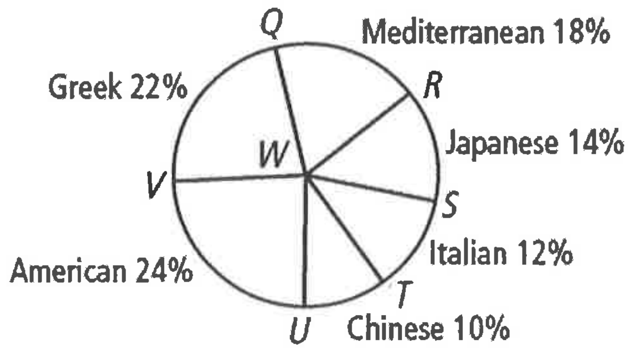
a. $\overset{\frown}{RS}$ $m\overset{\frown}{RS} = 110^\circ$

b. $\overset{\frown}{RTS}$ $m\overset{\frown}{RTS} = 360 - 110$
$= 250^\circ$

c. $\overset{\frown}{RST}$
$m\overset{\frown}{RST} = 180^\circ$

The circle graph shows the types of cuisine available in a city. Find $m\overset{\frown}{TRQ}$.

Types of Food



$TRQ = 12\% + 14\% + 18\% = 44\%$

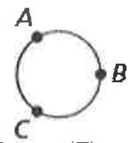
$m\overset{\frown}{TRQ} = (.44)(360) = 158.4^\circ$

Adjacent arcs are arcs of the same circle that intersect at exactly one point. *RS* and *ST* are adjacent arcs.

Postulate

Postulate 10.1 Arc Addition Postulate

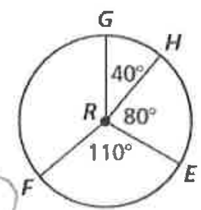
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.



$m\overset{\frown}{ABC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$

Find the measure of each arc.

a. $\overset{\frown}{GE}$ **b.** $\overset{\frown}{GEF}$ **c.** $\overset{\frown}{GF}$



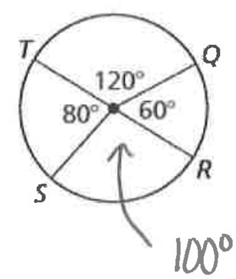
$\boxed{m\overset{\frown}{GE} = 40+80 = 120^\circ}$

$\boxed{m\overset{\frown}{GF} = 360-230 = 130^\circ}$

$\boxed{m\overset{\frown}{GEF} = 40+80+110^\circ = 230^\circ}$

Identify the given arc as a *major arc*, *minor arc*, or *semicircle*. Then find the measure of the arc.

1. $\overset{\frown}{TQ}$ **2.** $\overset{\frown}{QRT}$ **3.** $\overset{\frown}{TQR}$



Minor Arc

$m\overset{\frown}{TQ} = 120^\circ$

Major Arc

$m\overset{\frown}{QRT} = 60+100+80$
$= 240^\circ$

Semi-circle

$m\overset{\frown}{TQR} = 180^\circ$

4. $\overset{\frown}{QS}$ **5.** $\overset{\frown}{TS}$ **6.** $\overset{\frown}{RST}$

Minor Arc

$m\overset{\frown}{QS} = 60+100$
$= 160^\circ$

minor Arc

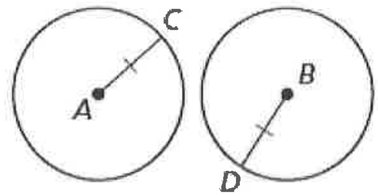
$m\overset{\frown}{TS} = 80^\circ$

Semi-circle

$m\overset{\frown}{RST} = 180^\circ$

Theorem 10.3 Congruent Circles Theorem

Two circles are congruent circles if and only if they have the same radius.



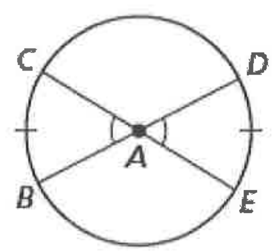
Proof Ex. 35, p. 544

$\odot A \cong \odot B$ if and only if $\overline{AC} \cong \overline{BD}$.

Arcs are congruent iff they have the same measure AND they are arcs of the same circle or congruent circles.

Theorem 10.4 Congruent Central Angles Theorem

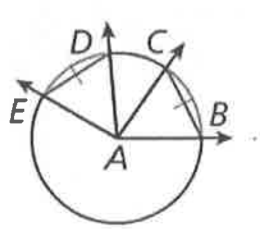
In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding central angles are congruent.



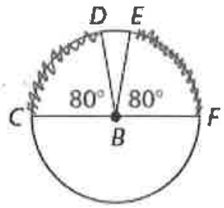
Proof Ex. 37, p. 544

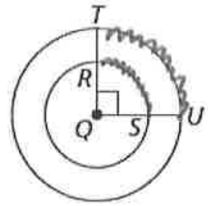
$\overset{\frown}{BC} \cong \overset{\frown}{DE}$ if and only if $\angle BAC \cong \angle DAE$.

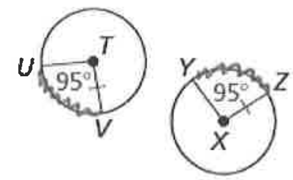
*** Congruent Arcs have congruent Chords



Tell whether the red arcs are congruent. Explain why or why not.

a.


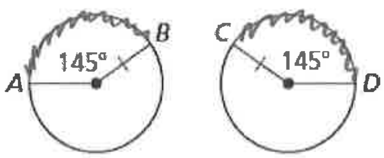
b.


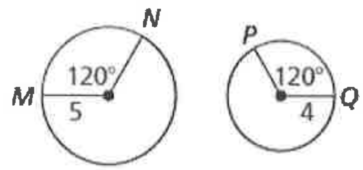
c.


$\overset{\frown}{CD} \cong \overset{\frown}{EF}$
they are of the same circle and their central ∠'s are ≅

$\overset{\frown}{RS}$ and $\overset{\frown}{TU}$
have same measure but are of two circles that aren't ≅ so they aren't ≅

$\overset{\frown}{UV} \cong \overset{\frown}{YZ}$
they are of ≅ circles are their central ∠'s are ≅

7.


8.


$\overset{\frown}{AB} \cong \overset{\frown}{CD}$

$\overset{\frown}{MN}$ and $\overset{\frown}{PQ}$ have same measure, but aren't ≅

Theorem 10.5 Similar Circles Theorem

All circles are similar.

Proof p. 541; Ex. 33, p. 544

Homework: pg. 542 #3-16, 18-24, 26, 29, 30