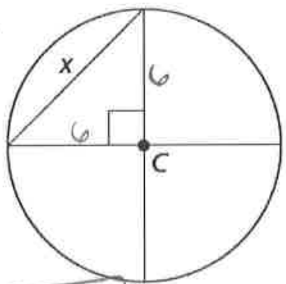
10.3 Using Chords

Bellwork:

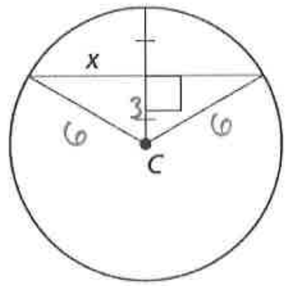
Find the value of *x* given that *C* is the center of the circle and that the circle has a diameter of 12.

1.



$\boxed{X = 6\sqrt{2}}$

2.



$3^2 + x^2 = 6^2$

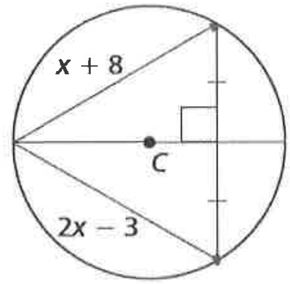
$9 + x^2 = 36$

$x^2 = 27$

$x = \sqrt{9}\sqrt{3}$

$\boxed{x = 3\sqrt{3}}$

3.

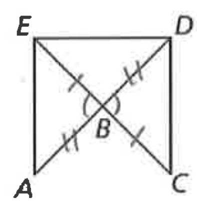


$x + 8 = 2x - 3$

$\boxed{11 = x}$

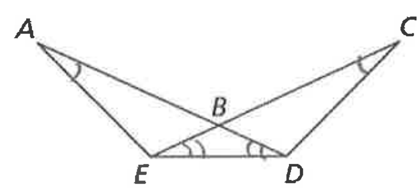
Write a proof.

1. Given: *B* is the midpoint of $\overline{EC}$ and $\overline{DA}$.
Prove: $\triangle AEB \cong \triangle DCB$



S	R
B mdpt of $\overline{EC}$ and $\overline{DA}$	Given
$\overline{EB} \cong \overline{BC}$, $\overline{AB} \cong \overline{BD}$	Def. of mdpt
$\angle EBA \cong \angle DBC$	Vert. ∠'s ≅ Thm
$\triangle AEB \cong \triangle DCB$	SAS

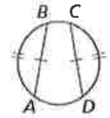
2. Given: $\angle BDE \cong \angle BED$, $\angle A \cong \angle C$
Prove: $\triangle AED \cong \triangle CDE$



S	R
$\angle BDE \cong \angle BED$, $\angle A \cong \angle C$	Given
$\overline{DE} \cong \overline{DE}$	Reflexive POC
$\triangle AED \cong \triangle CDE$	AAS

Theorem 10.6 Congruent Corresponding Chords Theorem

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

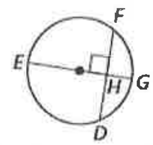


Proof Ex. 19, p. 550

$\overset{\frown}{AB} \cong \overset{\frown}{CD}$ if and only if $\overline{AB} \cong \overline{CD}$.

Theorem 10.7 Perpendicular Chord Bisector Theorem

If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

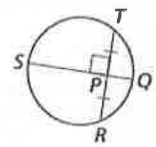


Proof Ex. 22, p. 550

If $\overline{EG}$ is a diameter and $\overline{EG} \perp \overline{DF}$, then $\overline{HD} \cong \overline{HF}$ and $\overset{\frown}{GD} \cong \overset{\frown}{GF}$.

Theorem 10.8 Perpendicular Chord Bisector Converse

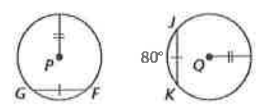
If one chord of a circle is a perpendicular bisector of another chord, then the first chord is a diameter.



Proof Ex. 23, p. 550

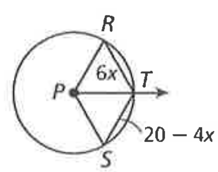
If $\overline{QS}$ is a perpendicular bisector of $\overline{TR}$, then $\overline{QS}$ is a diameter of the circle.

In the diagram, $\odot P \cong \odot Q$, $\overline{FG} \cong \overline{JK}$, and $m\overset{\frown}{JK} = 80°$. Find $m\overset{\frown}{FG}$.



$\boxed{m\overset{\frown}{FG} = 80°}$

$\overrightarrow{PT}$ bisects $\angle RPS$. Find RT.



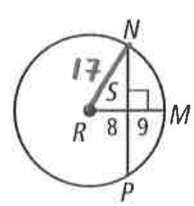
$6x = 20 - 4x$

$10x = 20$

$x = 2$

$\boxed{RT = 6(2) = 12}$

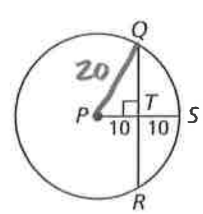
Find *NP*.



$NS = 15$ (8-15-17 △)

$\boxed{NP = 2(15) = 30}$

Find QR to the nearest tenth.



$QT = 10\sqrt{3}$ (30-60-90 △)

$\boxed{QR = 2(10\sqrt{3}) = 20\sqrt{3}}$

a. Find HK.

$\boxed{HK = 2(7) = 14}$

b. Find $m\widehat{HK}$.

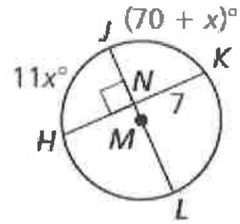
$11x = 70 + x$

$10x = 70$

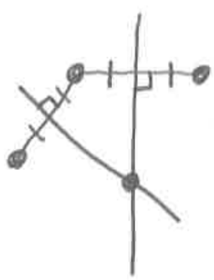
$x = 7$

$m\widehat{HJ} = 11(7) = 77^\circ$

$\boxed{m\widehat{HK} = 2(77) = 154^\circ}$



Three bushes are arranged in a garden, as shown. Where should you place a sprinkler so that it is the same distance from each bush?

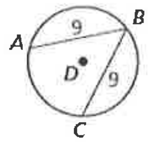


Find the ⊥ bisector between each bush. Where they meet is the center of the circle.

In Exercises 1 and 2, use the diagram of ⊙*D*.

1. If $m\overset{\frown}{AB} = 110°$, find $m\overset{\frown}{BC}$.

2. If $m\overset{\frown}{AC} = 150°$, find $m\overset{\frown}{AB}$.



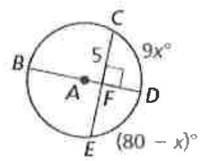
$\boxed{m\overset{\frown}{BC} = 110°}$

$m\overset{\frown}{AB} = \frac{360-150}{2} = \frac{210}{2} = \boxed{105°}$

In Exercises 3 and 4, find the indicated length or arc measure.

3. *CE* $= 2(5) = \boxed{10}$

4. $m\overset{\frown}{CE}$



$9x = 80 - x$

$10x = 80$

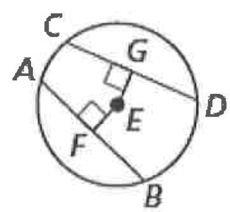
$x = 8$

$m\overset{\frown}{CD} = 9(8) = 72$

$\boxed{m\overset{\frown}{CE} = 2(72) = 144°}$

Theorem 10.9 Equidistant Chords Theorem

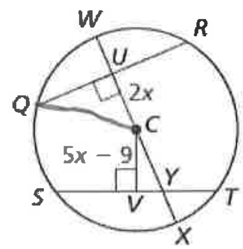
In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.



Proof Ex. 25, p. 550

$\overline{AB} \cong \overline{CD}$ if and only if $EF = EG$.

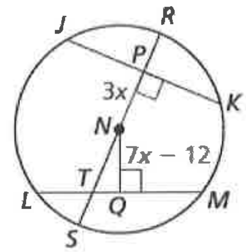
In the diagram, $QR = ST = 16$, $CU = 2x$, and $CV = 5x - 9$. Find the radius of $\odot C$.



$2x = 5x - 9$
$-3x = -9$
$x = 3$
$CU = 2(3) = 6$
$QU = 8$

$\boxed{QC = 10}$
(6-8-10 Triplet)

In the diagram, $JK = LM = 24$, $NP = 3x$, and $NQ = 7x - 12$. Find the radius of $\odot N$.



$3x = 7x - 12$
$-4x = -12$
$x = 3$
$NP = 3(3) = 9$
$JP = 12$
$\boxed{JN = 15}$ (3-4-5 Triplet)

Homework:

pg. 549 #3-10, 13-17

and WS 10.3 - Using Chords