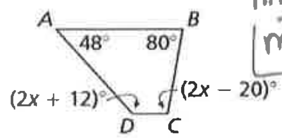
10.4 Inscribed Angles and Polygons

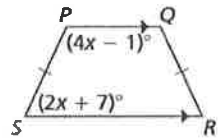
Bellwork:

Find the measure of each angle in the polygon.



$m\angle D = 2(60)+12$

$\boxed{m\angle D = 132°}$



$4x - 1 + 2x + 7 = 180$

$6x + 6 = 180$

$6x = 174$

$x = 29$

$48 + 80 + 2x + 12 + 2x - 20 = 360$

$4x + 120 = 360$

$4x = 240$

$x = 60$

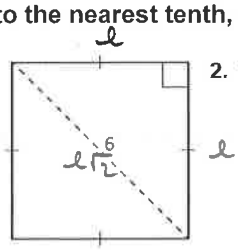
$m\angle C = 2(60) - 20$

$\boxed{m\angle C = 100°}$

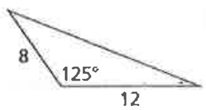
$\boxed{\begin{array}{l} m\angle P = m\angle Q = 4(29) - 1 = 115° \\ m\angle S = m\angle R = 2(29) + 7 = 65° \end{array}}$

Find the area of the geometric figure. Round your answer to the nearest tenth, when necessary.

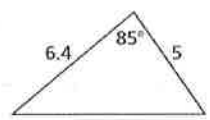
1.



2.



3.



$\ell\sqrt{2} = 6$

$\ell = \frac{6}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$

$\boxed{A = (3\sqrt{2})^2 = 9 \cdot 2 = 18}$

$A = \frac{1}{2}(8)(12)\sin 125$

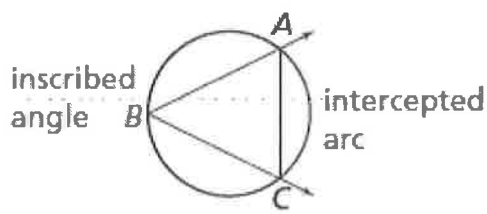
$\boxed{A = 39.3}$

$A = \frac{1}{2}(6.4)(5)\sin 85$

$\boxed{A = 15.9}$

Inscribed Angle and Intercepted Arc

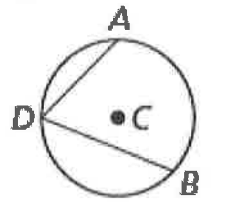
An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. An arc that lies between two lines, rays, or segments is called an **intercepted arc**. If the endpoints of a chord or arc lie on the sides of an inscribed angle, then the chord or arc is said to **subtend** the angle.



$\angle B$ intercepts $\overset{\frown}{AC}$.
$\overset{\frown}{AC}$ subtends $\angle B$.
$\overline{AC}$ subtends $\angle B$.

Theorem 10.10 Measure of an Inscribed Angle Theorem

The measure of an inscribed angle is one-half the measure of its intercepted arc.

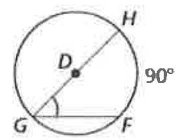


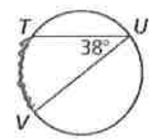
$m\angle ADB = \frac{1}{2} m\overset{\frown}{AB}$

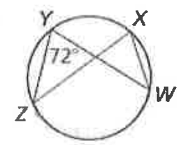
Proof Ex. 37, p. 560

Find the measure of the red arc or angle.

$m\angle HGF$ $m\overset{\frown}{TV}$ $m\angle WXZ$

1. 

2. 

3. 

$m\angle HGF = \frac{1}{2} m\overset{\frown}{HF}$

$m\angle HGF = \frac{1}{2}(90)$

$= \boxed{45°}$

$m\overset{\frown}{TV} = 2m\angle TUV$

$m\overset{\frown}{TV} = 2(38)$

$= \boxed{76°}$

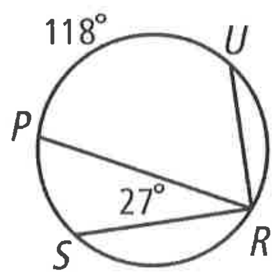
$m\overset{\frown}{WZ} = 2m\angle WYZ$

$m\overset{\frown}{WZ} = 2(72) = 144°$

$m\angle WXZ = \frac{1}{2}(m\overset{\frown}{WZ})$

$= \frac{1}{2}(144) = \boxed{72°}$

Find each measure.



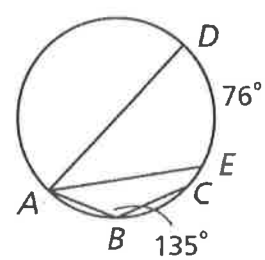
m∠PRU

$m\angle PRU = \frac{1}{2}(118) = 59°$

$m\overset{\frown}{SP}$

$m\overset{\frown}{SP} = 2(27) = 54°$

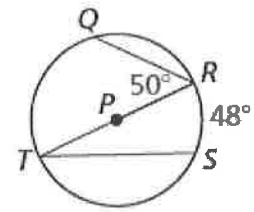
Find each measure.



$m\overset{\frown}{ADC}$ $m\overset{\frown}{ADC} = 2(135) = 270°$

m∠DAE $m\angle DAE = \frac{1}{2}(76) = 38°$

Find the indicated measure.



a. $m\angle T$ $= \frac{1}{2}(48) = 24°$

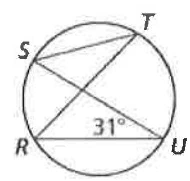
b. $m\widehat{QR}$

$m\widehat{RT} = 180°$

$m\widehat{QT} = 2(50) = 100°$

$m\widehat{QR} = 180 - 100 = 80°$

Find $m\widehat{RS}$ and $m\angle STR$. What do you notice about $\angle STR$ and $\angle RUS$?



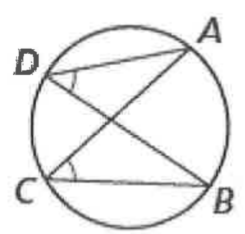
$m\widehat{RS} = 2(31) = 62°$

$m\angle STR = \frac{1}{2}(62) = 31°$

$\angle STR \cong \angle RUS$

Theorem 10.11 Inscribed Angles of a Circle Theorem

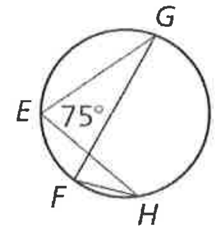
If two inscribed angles of a circle intercept the same arc, then the angles are congruent.



$\angle ADB \cong \angle ACB$

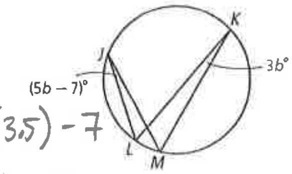
Proof Ex. 38, p. 560

Given $m\angle E = 75°$, find $m\angle F$.



$m\angle F = 75°$

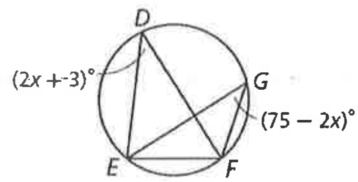
Find m∠*LJM*.



$5b - 7 = 3b$
$-7 = -2b$
$3.5 = b$

$m\angle LJM = 5(3.5) - 7$
$= 17.5 - 7$
$= \boxed{10.5}$

Find m∠*EDF*.

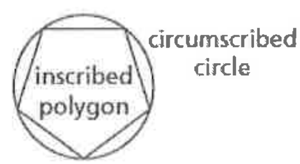


$2x + 3 = 75 - 2x$
$4x = 72$
$x = 18$

$m\angle EDF =$
$2(18) + 3 = \boxed{39°}$

Inscribed Polygon

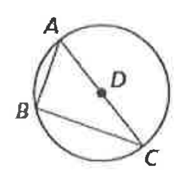
A polygon is an **inscribed polygon** when all its vertices lie on a circle. The circle that contains the vertices is a **circumscribed circle**.



Theorems

Theorem 10.12 Inscribed Right Triangle Theorem

If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.

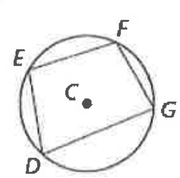


$m\angle ABC = 90°$ if and only if $\overline{AC}$ is a diameter of the circle.

Proof Ex. 39, p. 560

Theorem 10.13 Inscribed Quadrilateral Theorem

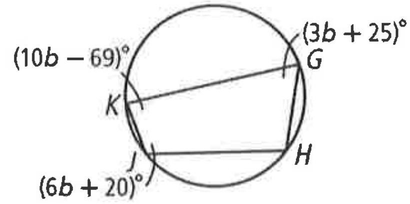
A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.



D, E, F, and G lie on $\odot C$ if and only if $m\angle D + m\angle F = m\angle E + m\angle G = 180°$.

Proof Ex. 40, p. 560

Find the angle measures of *GHJK*.



$3b + 25 + 6b + 20 = 180$

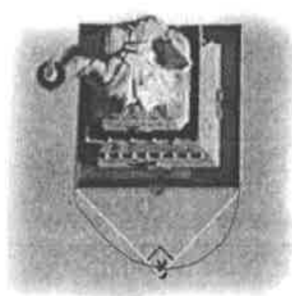
$9b + 45 = 180$

$9b = 135$

$b = 15$

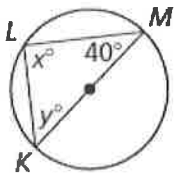
$$\boxed{\begin{aligned} m\angle K &= 10(15) - 69 = 81° \\ m\angle H &= 180 - 81 = 99° \\ m\angle J &= 6(15) + 20 = 110° \\ m\angle G &= 180 - 110 = 70° \end{aligned}}$$

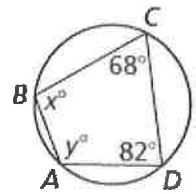
Your camera has a 90° field of vision, and you want to photograph the front of a statue. You stand at a location in which the front of the statue is all that appears in your camera's field of vision, as shown. You want to change your location. Where else can you stand so that the front of the statue is all that appears in your camera's field of vision?

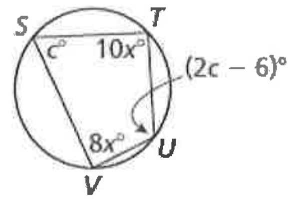


Anywhere on the semi-circle.

Find the value of each variable.

4. 

5. 

6. 

$x = 90°$

$y = 90 - 40 = 50°$

$y = 180 - 68 = 112°$

$x = 180 - 82 = 98°$

$10x + 8x = 180$

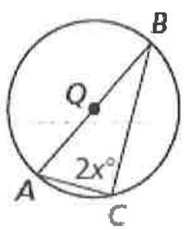
$18x = 180$

$x = 10$

$c + 2c - 6 = 180$

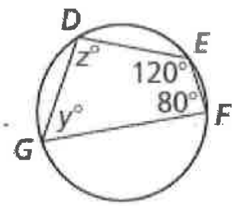
$3c = 186$

$c = 62$

a. 

$2x = 90$

$x = 45$

b. 

$y = 180 - 120 = 60°$

$z = 180 - 80 = 100°$

Homework:

pg. 558 #3-8, 11-16, 19-21

AND

WS 10.4 - Inscribed Angles and Polygons