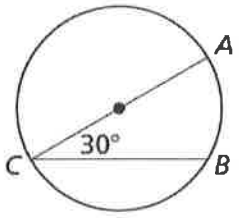
10.5 Angle Relationships in Circles

Bellwork

Find the indicated measure.

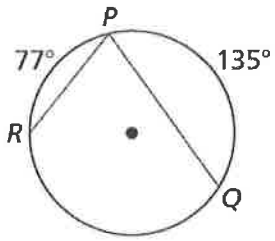
1. $m\overset{\frown}{BC}$



$m\overset{\frown}{AB} = 60°$

$m\overset{\frown}{BC} = 180 - 60 = 120°$

2. $m\angle P$



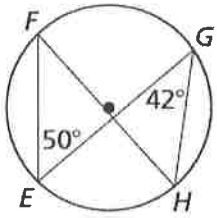
$m\overset{\frown}{RQ} = 360 - (135 + 77)$

$= 148°$

$m\angle P = \frac{1}{2}(148)$

$= 74°$

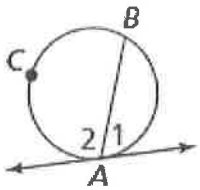
3. $m\overset{\frown}{EH}$



$m\overset{\frown}{EH} = 2(42) = 84°$

Theorem 10.14 Tangent and Intersected Chord Theorem

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one-half the measure of its intercepted arc.

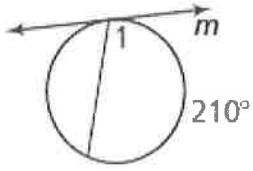


Proof Ex. 33, p. 568

$m\angle 1 = \frac{1}{2}m\overset{\frown}{AB}$ $m\angle 2 = \frac{1}{2}m\overset{\frown}{BCA}$

Line *m* is tangent to the circle. Find the indicated measure.

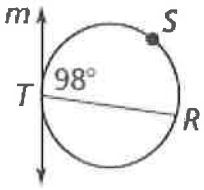
1. $m\angle 1$



$m\angle 1 = \frac{1}{2}(210)$

$= 105°$

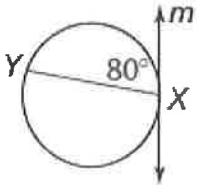
2. $m\overset{\frown}{RST}$



$m\overset{\frown}{RST} = 2(98)$

$= 196°$

3. $m\overset{\frown}{XY}$

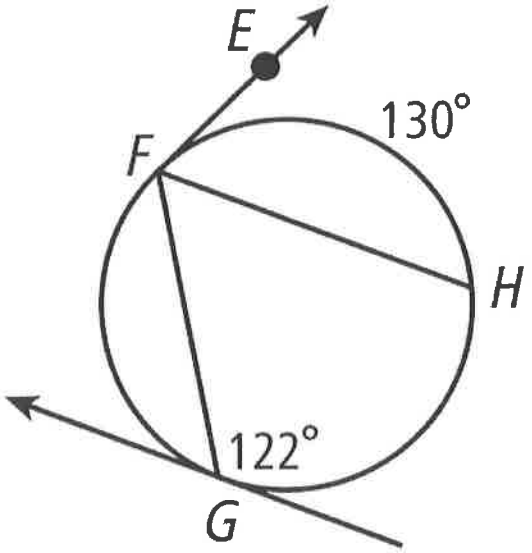


$m\overset{\frown}{XY} = 2(80) = 160°$

Find each measure.

m∠*EFH*

$m\angle EFH = \frac{1}{2}(130) = 65°$



m$\overset{\frown}{GF}$

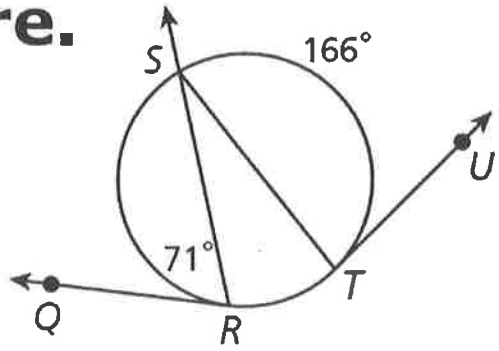
$180 - 122 = 58°$

$m\overset{\frown}{GF} = 2(58) = 116°$

Find each measure.

m∠*STU*

$m\angle STU = \frac{1}{2}(166) = 83°$



m$\overset{\frown}{SR}$

$m\overset{\frown}{SR} = 2(71) = 142°$

Core Concept

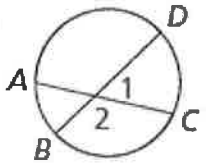
Intersecting Lines and Circles

If two nonparallel lines intersect a circle, there are three places where the lines can intersect.

on the circle | inside the circle | outside the circle

Theorem 10.15 Angles Inside the Circle Theorem

If two chords intersect *inside* a circle, then the measure of each angle is one-half the *sum* of the measures of the arcs intercepted by the angle and its vertical angle.



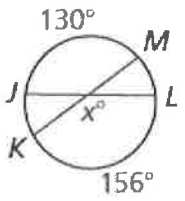
$m\angle 1 = \frac{1}{2}(m\overset{\frown}{DC} + m\overset{\frown}{AB})$,

$m\angle 2 = \frac{1}{2}(m\overset{\frown}{AD} + m\overset{\frown}{BC})$

Proof Ex. 35, p. 568

Find the value of the variable.

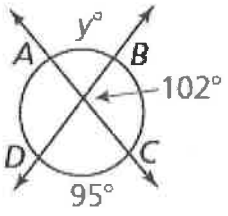
a.



$x = \frac{1}{2}(130+156)$

$= \frac{1}{2}(286)$

$= 143°$



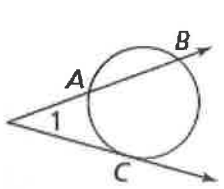
$102 = \frac{1}{2}(95+y)$

$204 = 95 + y$

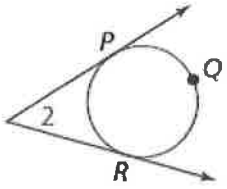
$109 = y$

Theorem 10.16 Angles Outside the Circle Theorem

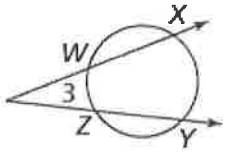
If a tangent and a secant, two tangents, or two secants intersect *outside* a circle, then the measure of the angle formed is one-half the *difference* of the measures of the intercepted arcs.



$m\angle 1 = \frac{1}{2}(m\overset{\frown}{BC} - m\overset{\frown}{AC})$



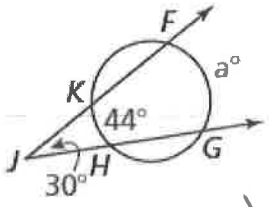
$m\angle 2 = \frac{1}{2}(m\overset{\frown}{PQR} - m\overset{\frown}{PR})$



$m\angle 3 = \frac{1}{2}(m\overset{\frown}{XY} - m\overset{\frown}{WZ})$

Proof Ex. 37, p. 568

Find the value of the variable.

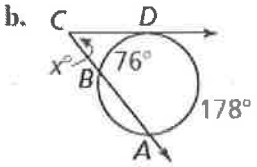


$30 = \frac{1}{2}(a - 44)$

$60 = a - 44$

$104 = a$

b.

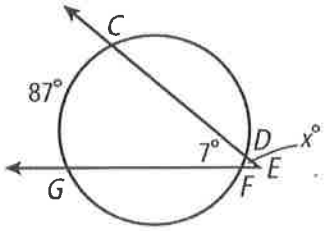


$X = \frac{1}{2}(178 - 76)$

$X = 51$

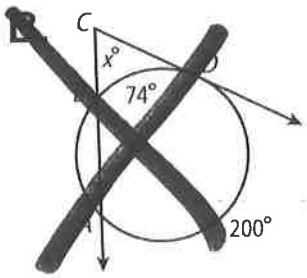
Find the value of *x*.

A.



$X = \frac{1}{2}(87 - 7)$

$X = 40$



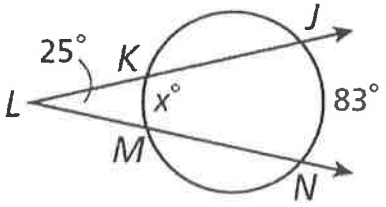
Find the value of *x*.

$25 = \frac{1}{2}(83 - X)$

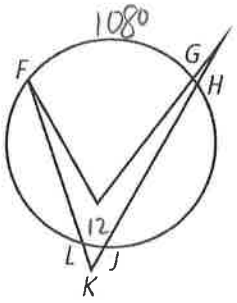
$50 = 83 - X$

$-33 = -X$

$33 = X$



In the company logo shown, $m\overset{\frown}{FH} = 108°$, and $m\overset{\frown}{LJ} = 12°$. What is $m\angle FKH$?

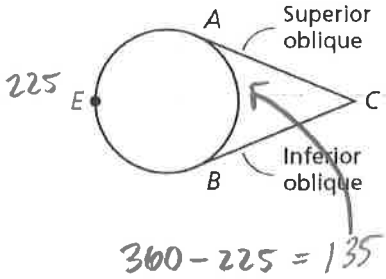


$m\angle K = \frac{1}{2}(108 - 12)$

$= \frac{1}{2}(96)$

$= 48°$

Two of the six muscles that control eye movement are attached to the eyeball and intersect behind the eye. If $m\overset{\frown}{AEB} = 225°$, what is $m\angle ACB$?

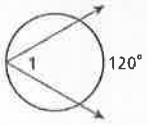
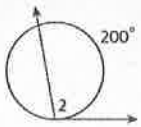
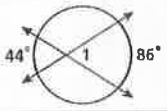
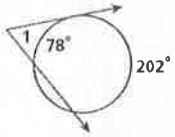
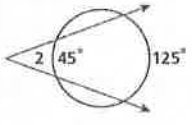


$m\angle C = \frac{1}{2}(225 - 135)$

$= \frac{1}{2}(90)$

$m\angle C = 45°$

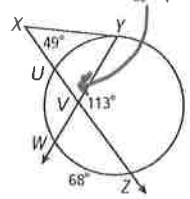
Angle Relationships in Circles

VERTEX OF THE ANGLE	MEASURE OF ANGLE	DIAGRAMS	
On a circle	Half the measure of its intercepted arc	 $m\angle 1 = 60°$	 $m\angle 2 = 100°$
Inside a circle	Half the sum of the measures of its intercepted arcs		$m\angle 1 = \frac{1}{2}(44° + 86°)$ $= 65°$
Outside a circle	Half the difference of the measures of its intercepted arcs	 $m\angle 1 = \frac{1}{2}(202° - 78°)$ $= 62°$	 $m\angle 2 = \frac{1}{2}(125° - 45°)$ $= 40°$

*** On the center of the circle: Angle = Arc

$180 - 113 = 67$

Find $m\overset{\frown}{YZ}$.



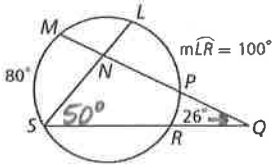
$m\angle XYV = 180 - (49 + 67)$

$= 64^\circ$

$m\overset{\frown}{YW} = 2(64) = 128$

$m\overset{\frown}{YZ} = 128 + 68 = \boxed{196^\circ}$

Find $m\overset{\frown}{LP}$



$m\angle S = \frac{1}{2}(100) = 50$

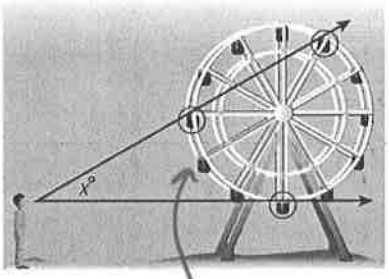
$26 = \frac{1}{2}(80 - \overset{\frown}{PR})$ $m\overset{\frown}{PR} = 28$

$52 = 80 - \overset{\frown}{PR}$ $m\overset{\frown}{LP} = 100 - 28 = \boxed{72^\circ}$

$-28 = -\overset{\frown}{PR}$

3. An observer watches people riding a Ferris wheel that has 12 equally spaced cars.

Find x.



$4(30)$

$3(30)$

$\frac{360}{12} = 30^\circ$

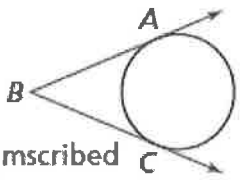
$X = \frac{1}{2}(120 - 90)$

$X = \frac{1}{2}(30) = 15^\circ$

Core Concept

Circumscribed Angle

A **circumscribed angle** is an angle whose sides are tangent to a circle.

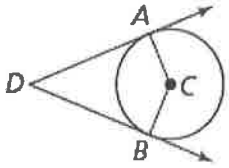


circumscribed angle

Theorem

Theorem 10.17 Circumscribed Angle Theorem

The measure of a circumscribed angle is equal to 180° minus the measure of the central angle that intercepts the same arc.



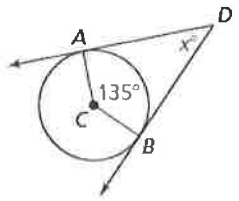
Proof Ex. 38, p. 568

$m\angle ADB = 180^\circ - m\angle ACB$

* This is because $m\angle A$ and $m\angle B = 90^\circ$

Find the value of x.

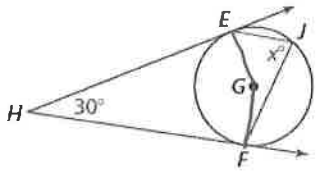
a.



$X = 180 - 135$

$X = 45°$

b.

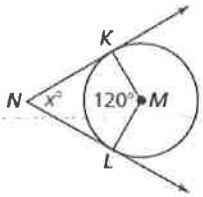


$m\angle EGF = 180 - 30 = 150°$

$m\overset{\frown}{EF} = 150°$

$X = \frac{1}{2}(150) = 75°$

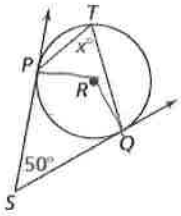
6.



$X = 180 - 120$

$X = 60°$

7.

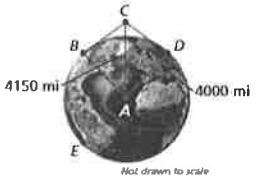


$m\angle PRQ = 180 - 50 = 130$

$m\overset{\frown}{PQ} = 130°$

$X = \frac{1}{2}(130) = 65°$

The northern lights are bright flashes of colored light between 50 and 200 miles above Earth. A flash occurs 150 miles above Earth at point C. What is the measure of $\overset{\frown}{BD}$, the portion of Earth from which the flash is visible? (Earth's radius is approximately 4000 miles.)



$m\angle CAD = \cos^{-1}\left(\frac{4000}{4150}\right) = 15.45°$

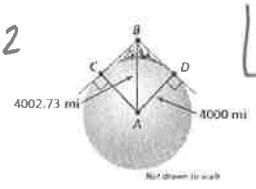
$m\overset{\frown}{BD} = 30.9°$

$m\angle BAD = 2(15.45) = 30.9°$

8. You are on top of Mount Rainier on a clear day. You are about 2.73 miles above sea level at point B. Find $m\overset{\frown}{CD}$, which represents the part of Earth that you can see.

$m\angle BAD = \cos^{-1}\left(\frac{4000}{4002.73}\right) = 2.12$

$m\overset{\frown}{CD} = 4.24$



$m\angle CAD = 2(2.12) = 4.24$

Homework:
pg. 566 #4-14, 17-26, 39-40
WS 10.5 Angle Relationships in Circles

Homework by Topic:

Tangent and Intersected Chord Theorem:

#3-6

Angles Inside/Outside the Circle Theorem:

#7-12, 15,16,26,39,40,

Circumscribed Angle Theorem:

#13, 14, 17-24