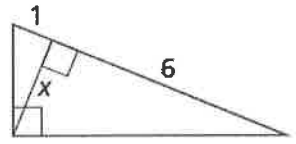
10.6 Segment Relationships in Circles

Bellwork

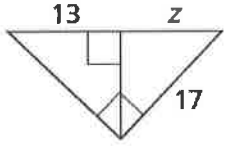
Find the value of the variable.

1. 

$x^2 = 1 \cdot 6$

$x^2 = 6$

$x = \sqrt{6}$

2. 

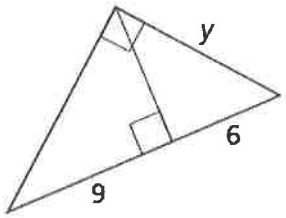
$17^2 = z(z+13)$

$289 = z^2 + 13z$

$0 = z^2 + 13z - 289$

$x = \frac{-13 \pm \sqrt{13^2 - 4(1)(-289)}}{2}$

$x = \frac{-13 \pm \sqrt{1325}}{2} = 11.7$

3. 

$y^2 = 6(15)$

$\sqrt{y^2} = \sqrt{90}$

$y = 3\sqrt{10}$

More Warm Up

Solve the equation.

1. $x(x + 5) = (x + 1)(x + 2)$

$x^2 + 5x = x^2 + 2x + x + 2$

$5x = 3x + 2$

$2x = 2$

$x = 1$

2. $2x(5) = (2x + 3)(x + 1)$

$10x = 2x^2 + 2x + 3x + 3$

$0 = 2x^2 - 5x + 3$

$2x^2 - 2x - 3x + 3$

$2x(x-1) - 3(x-1)$

$(2x-3)(x-1) = 0$

$x = \frac{3}{2}, \; x = 1$

$2 \cdot 3 = 6$

1	6
-2	-3

3. $(x + 2)(x + 5) = 2x(x + 2)$

$x^2 + 5x + 2x + 10 = 2x^2 + 4x$

$x^2 + 7x + 10 = 2x^2 + 4x$

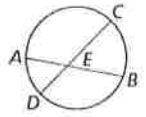
$0 = x^2 - 3x - 10$

$0 = (x-5)(x+2)$

$x = 5, \; -2$

Theorem 10.18 Segments of Chords Theorem

If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



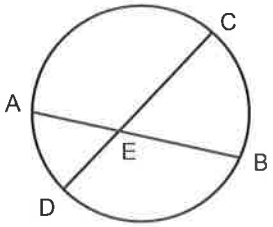
$EA \cdot EB = EC \cdot ED$

Proof Ex. 19, p. 574

part•part=part•part

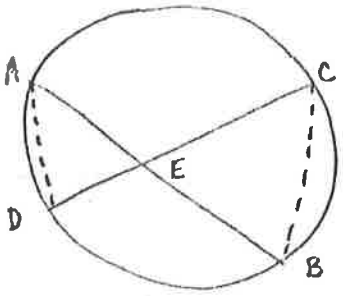
Proof of Segments of Chords Theorem

Theorem Proof



1. Draw chord AC and chord DB.
2. Show that Triangles AEB and DEB are similar.
3. Use the proportional side lengths to set up an equation.

Proof:



$\angle AED \cong \angle BEC$ by Vert. $\angle$'s Theorem

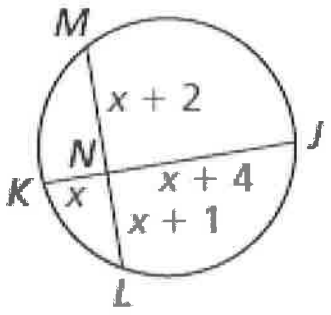
$\angle DAE \cong \angle BCE$ because they subtend the same arc

$\triangle DAE \sim \triangle BCE$ by AA

$\frac{EA}{EC} = \frac{ED}{EB}$ Parts of Similar $\triangle$'s are proportional

$EA \cdot EB = EC \cdot ED$ by Cross Multiplication

Find *ML* and *JK*.



$(x+2)(x+1) = x(x+4)$

$x^2 + 1x + 2x + 2 = x^2 + 4x$

$3x + 2 = 4x$

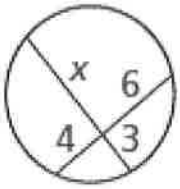
$2 = x$

$ML = (2+2) + (2+1) = 4+3 = 7$

$JK = 2 + (2+4) = 8$

Find the value of *x*.

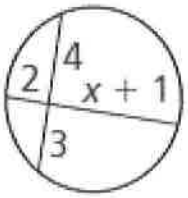
1.



$3x = 24$

$x = 8$

2.



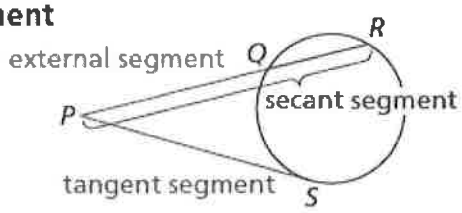
$2x + 2 = 12$

$2x = 10$

$x = 5$

Tangent Segment and Secant Segment

A **tangent segment** is a segment that is tangent to a circle at an endpoint. A **secant segment** is a segment that contains a chord of a circle and has exactly one endpoint outside the circle. The part of a secant segment that is outside the circle is called an **external segment.**

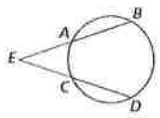


$\overline{PS}$ is a tangent segment.
$\overline{PR}$ is a secant segment.
$\overline{PQ}$ is the external segment of $\overline{PR}$.

Theorem 10.19 Segments of Secants Theorem

If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.

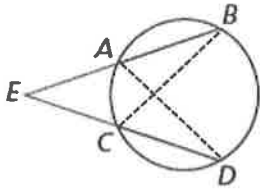
Proof Ex. 20, p. 574



$EA \cdot EB = EC \cdot ED$

outside • whole = outside • whole

Proof



1. Angles DEA and BEC are congruent
2. Angles ABC and ADC are congruent
3. Triangles are similar.
4. Set up proportional sides equation

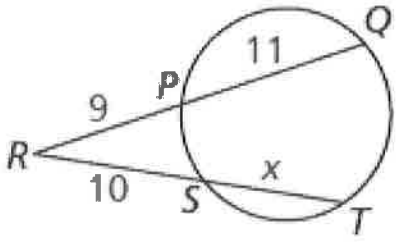
Find the value of x.

$9 \cdot 20 = 10(10+x)$

$180 = 100 + 10x$

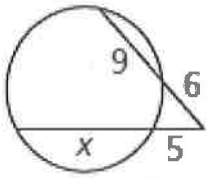
$80 = 10x$

$\boxed{8 = x}$



Find the value of x.

3.



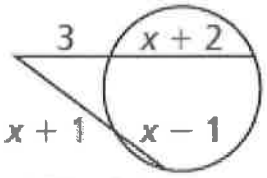
$5(5+x) = 6(15)$

$25 + 5x = 90$

$5x = 65$

$\boxed{x = 13}$

4.



$3(x+5) = (x+1)(2x)$

$3x + 15 = 2x^2 + 2x$

$0 = 2x^2 - x - 15$ $\quad 2(-15) = -30$

$0 = 2x^2 - 6x + 5x - 15$

$2x(x-3) + 5(x-3)$

1	30
2	15
3	10
5	-6

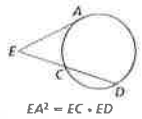
$0 = (2x+5)(x-3)$

$\boxed{x = -\frac{5}{2}, 3}$

Theorem 10.20 Segments of Secants and Tangents Theorem

If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.

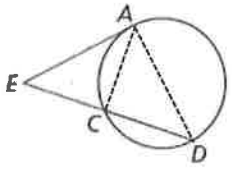
Proof Exs. 21 and 22, p. 574



$EA^2 = EC \cdot ED$

(tangent)^2 = outside • whole

Proof



1. Prove Triangles EAC and EDA are similar by showing:

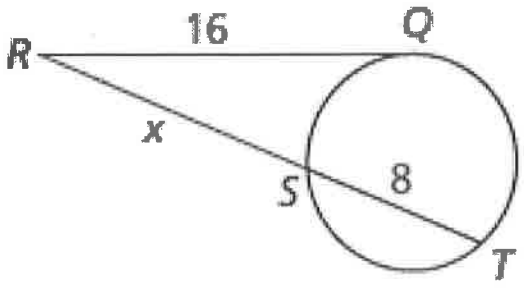
   a. Angle E is congruent to itself

   b. Angle ADC is half of arc AC and Angle EAC is half of arc AC. By the transitive property, these two angles are congruent.

   c. Set up proportional sides: EA/ED = EC/EA

   d. Cross multiply

Find *RS*.



$16^2 = x(x+8)$

$256 = x^2 + 8x$

$0 = x^2 + 8x - 256$

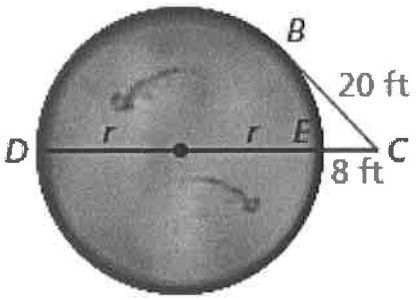
$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-8 \pm \sqrt{(8)^2-4(1)(-256)}}{2} = \frac{-8 \pm \sqrt{1088}}{2}$

$\frac{-8+\sqrt{1088}}{2} = 12.49$

$\frac{-8-\sqrt{1088}}{2} = -20.49$

Find the radius of the aquarium tank.



$(20)^2 = 8(8+2r)$

$400 = 64 + 16r$

$336 = 16r$

$21 = r$

8. WHAT IF? In Example 4, *CB* = 35 feet and *CE* = 14 feet. Find the radius of the tank.

$(35)^2 = 14(14+2r)$

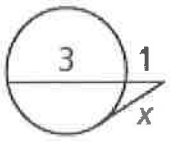
$1225 = 196 + 28r$

$1029 = 28r$

$36.75 = r$

Find the value of x.

5.

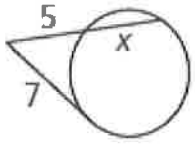


$x^2 = 1(4)$

$x^2 = 4$

$x = 2$

6.



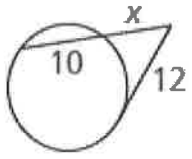
$7^2 = 5(5+x)$

$49 = 25 + 5x$

$24 = 5x$

$\frac{24}{5} = x$

7.



$12^2 = x(10+x)$

$144 = 10x + x^2$

$0 = x^2 + 10x - 144$

$(x+18)(x-8)$

$x = -18, 8$

$x = 8$

144

1	144
2	72
3	48
4	36
6	24
8	18

Homework:

pg. 573 #3-17, 25