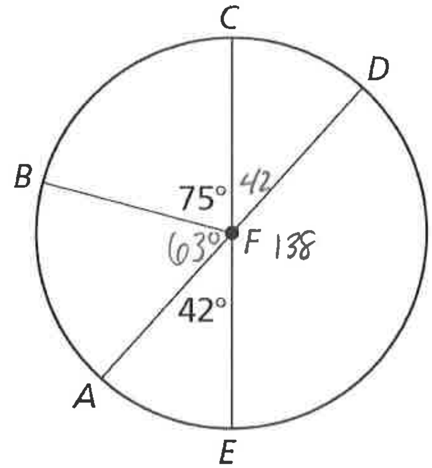
10.7 - Circles in the Coordinate Plane

Find the measure of the arc.

1. $\overset{\frown}{AB}$ = 63°
2. $\overset{\frown}{CD}$ = 42°
3. $\overset{\frown}{DE}$ = 138°
4. $\overset{\frown}{BCD}$ = 117°
5. $\overset{\frown}{AED}$ = 180°



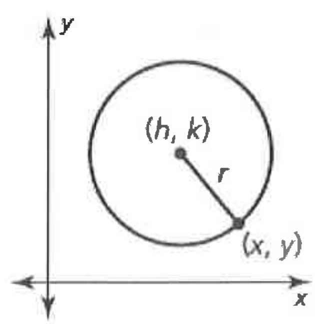
Consider a circle with radius *r and center* (*h*, *k*). Write the Distance Formula to represent
the distance *d* between a point (*x*, *y*) on the circle and
the center (*h*, *k*) of the circle. Then square each side of
the Distance Formula equation.

$d = \sqrt{(x-h)^2 + (y-k)^2}$

↓

$r = \sqrt{(x-h)^2 + (y-k)^2}$

$\boxed{r^2 = (x-h)^2 + (y-k)^2}$

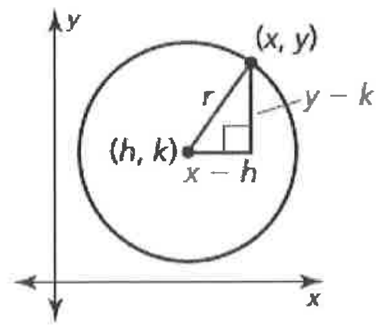


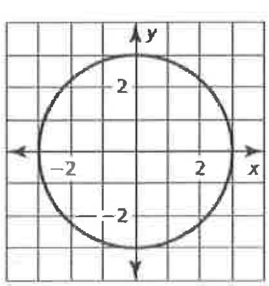
Standard Equation of a Circle

Let (x, y) represent any point on a circle with center (h, k) and radius r. By the Pythagorean Theorem (Theorem 9.1),

$$(x - h)^2 + (y - k)^2 = r^2.$$

This is the **standard equation of a circle** with center (h, k) and radius r.





Write the standard equation of each circle.

a. the circle shown at the left

center : (0,0) radius = 3

$(x)^2 + (y)^2 = 9$

b. a circle with center (0, −9) and radius 4.2

$(x-0)^2 + (y+9)^2 = (4.2)^2$

$x^2 + (y+9)^2 = 17.64$

Write the standard equation of the circle with the given center and radius.

center: (0, 0), radius: 2.5

$$x^2 + y^2 = 6.25$$

center: (−2, 5), radius: 7

$$(x+2)^2 + (y-5)^2 = 49$$

center (−1, 3) and radius 5.

$$(x+1)^2 + (y-3)^2 = 25$$

The point (−5, 6) is on a circle with

center (−1, 3). Write the standard equation of the circle.

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$r = \sqrt{(-5-1)^2 + (6-3)^2}$$

$$r = \sqrt{(-6)^2 + (3)^2}$$

$$r = \sqrt{36 + 9}$$

$$r = \sqrt{45}$$

$$\boxed{(x+1)^2 + (y-3)^2 = 45}$$

The equation of a circle is $x^2 + y^2 - 8x + 4y - 16 = 0$. Find the center and the radius of the circle. Then graph the circle.

$x^2 - 8x + \underline{16} + y^2 + 4y + \underline{+4} = 16 + 16 + 4$

$(x-4)^2 + (y+2)^2 = 36$

center $(4,-2)$ $r=6$

3. The point (3, 4) is on a circle with center (1, 4). Write the standard equation of the circle.

$\boxed{(x-1)^2 + (y-4)^2 = 4}$

$d = \sqrt{(3-1)^2 + (4-4)^2}$

$d = \sqrt{(2)^2} = 2$

$r = 2$

4. The equation of a circle is $x^2 + y^2 - 8x + 6y + 9 = 0$. Find the center and the radius of the circle. Then graph the circle.

$x^2 - 8x + \underline{16} + y^2 + 6y + \underline{9} = -9 + 16 + 9$

$(x-4)^2 + (y+3)^2 = 16$

center $(4,-3)$ $r = 4$

Prove or disprove that the point $\left(\sqrt{2},\ \sqrt{2}\right)$ lies on the circle centered at the origin and containing the point (2, 0).

$d = \sqrt{(2-0)^2 + (0-0)} = \sqrt{(2)^2} = 2$

$r = 2$

$(x)^2 + (y)^2 = 4$

$(\sqrt{2})^2 + (\sqrt{2})^2 = 4$

$2 + 2 = 4$ ✓

$(\sqrt{2}, \sqrt{2})$ is on the circle

5. Prove or disprove that the point $\left(1,\ \sqrt{5}\right)$ lies on the circle centered at the origin and containing the point (0, 1).

$d = \sqrt{(0-0)^2 + (1-0)^2}$

$d = \sqrt{(1)^2} = 1$

$r = 1$

$x^2 + y^2 = 1$

$(1)^2 + (\sqrt{5})^2 = 1$

$1 + 5 = 1$

$6 \neq 1$

Not on the circle

The epicenter of an earthquake is the point on Earth's surface directly above the earthquake's origin. A seismograph can be used to determine the distance to the epicenter of an earthquake. Seismographs are needed in three different places to locate an earthquake's epicenter.

Use the seismograph readings from locations *A*, *B*, and *C* to find the epicenter of an earthquake.

- The epicenter is 7 miles away from $A(-2, 2.5)$.
- The epicenter is 4 miles away from $B(4, 6)$.
- The epicenter is 5 miles away from $C(3, -2.5)$.

6. Why are three seismographs needed to locate an earthquake's epicenter?

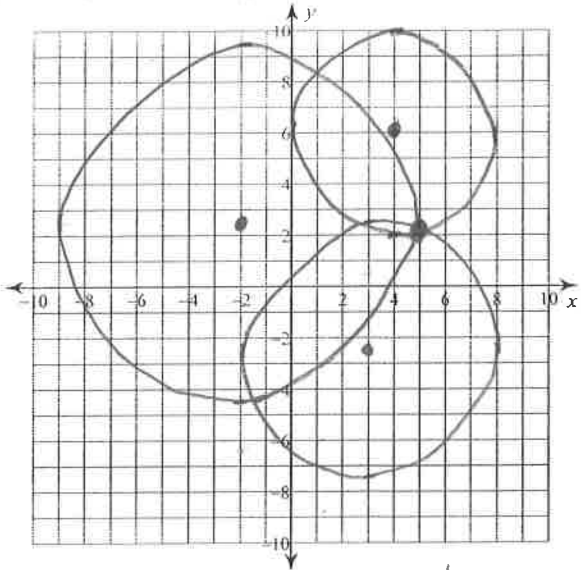
*** Teacher Notes: Need graph paper for this problem if you do it.

Homework:
pg. 579 #3-22, 29-32

**Cut back assignment as needed.

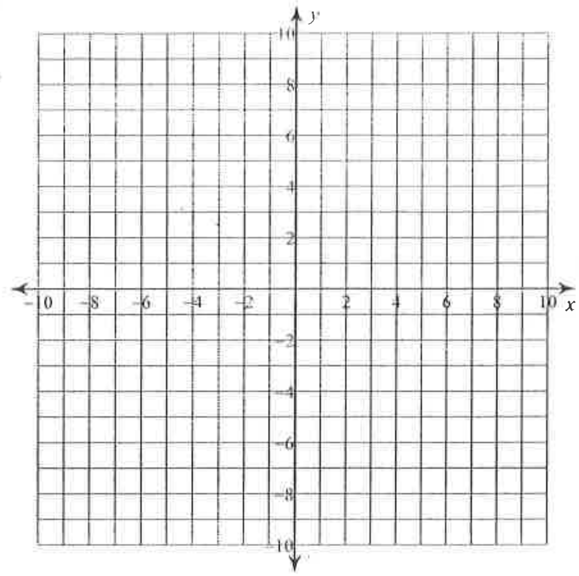
Graph Paper

1)

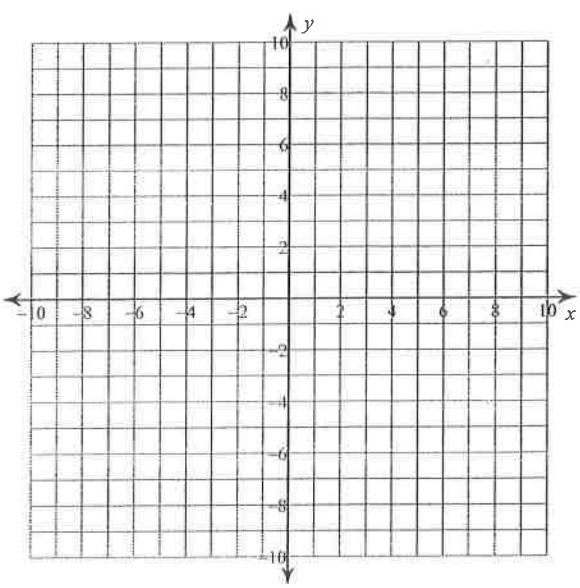


The epicenter is at about (5,2)

2)



3)



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