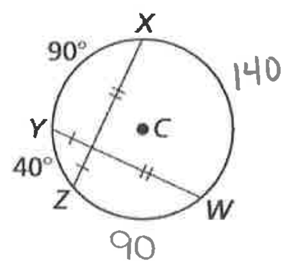
# 11.1 - Circumference and Arc Length

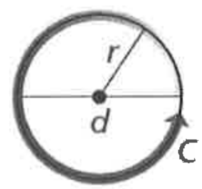
Find $m\widehat{WZY}$

$m\widehat{WZY} = 130°$




**Circumference of a Circle**

The circumference $C$ of a circle is $C = \pi d$ or $C = 2\pi r$, where $d$ is the diameter of the circle and $r$ is the radius of the circle.




$C = \pi d = 2\pi r$

**Find each indicated measure. Leave answer in terms of π (exact) and rounded to the nearest hundredth.**

**a.** circumference of a circle with a radius of 9 centimeters

$C = 2\pi(9)$

$\boxed{C = 18\pi \text{ cm} \quad \text{or} \quad 56.55 \text{ cm}}$

**b.** radius of a circle with a circumference of 26 meters

$C = 2\pi r$

$\frac{26}{2\pi} = \frac{2\pi r}{2\pi}$

$\boxed{\frac{13}{\pi} \text{ m} = r \quad \text{or} \quad 4.14 \text{ m}}$

**c.** Find the circumference of a circle with a diameter of 6 inches

$r = 3$

$C = 2\pi(3)$

$\boxed{C = 6\pi \text{ in} \quad \text{or} \quad 18.85 \text{ in}}$

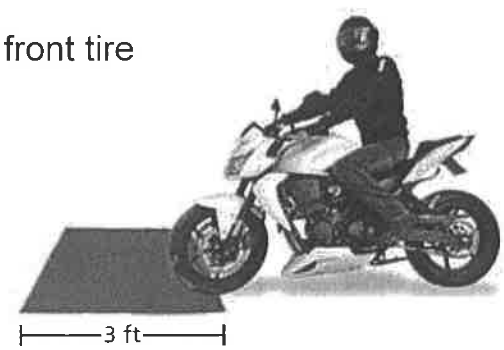
**d.** Find the diameter of a circle with a circumference of 17π feet.

$C = \pi d$

$\frac{17\cancel{\pi}}{\cancel{\pi}} = \frac{\cancel{\pi} d}{\cancel{\pi}}$

$\boxed{17 = d}$

The rider is attempting to stop with the front tire of the motorcycle in the painted rectangular box for a skills test.




The front tire makes exactly one-half additional revolution before stopping. The diameter of the tire is 25 inches. Is the front tire still in contact with the painted box? Explain.

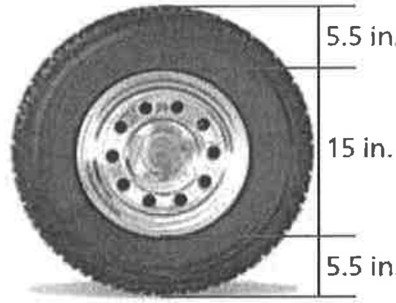
$C = \pi d$

$C = 25\pi$ or $78.54$ inches (which means the bike will travel 78.54 inches with each revolution)

Half a revolution $= \frac{1}{2}(78.54) = 39.27$ inches

No, the tire has gone past the box by 3.27 inches.

The dimensions of a car tire are shown. To the nearest foot, how far does the tire travel when it makes 15 revolutions?




One revolution:

$C = \pi d$

$C = \pi(26)$ or $26\pi$ or $81.68$ in

$81.68 \text{ in} \div 12 = 6.81$ feet

15 Revolutions: $6.81(15) = \boxed{102.15 \text{ feet}}$

A car tire has a diameter of 28 inches. How many revolutions does the tire make while traveling 500 feet?

$C = \pi d$

$C = \pi(28)$

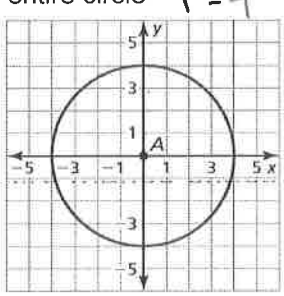
$C = 28\pi$ in

$\frac{28\pi}{12} = \frac{7\pi}{3}$ feet

$500 \div \frac{7\pi}{3} = 68.21$ Revolutions

Find the length of each red circular arc.

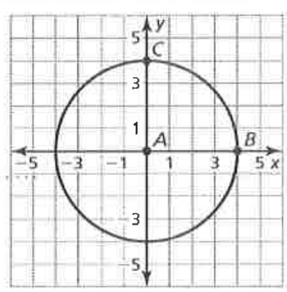
**a.** entire circle $r = 4$




$C = 2\pi(4)$

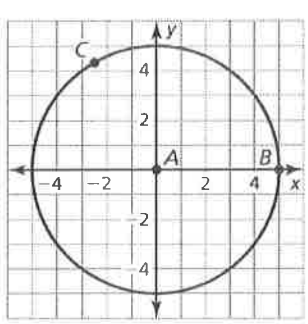
$C = 8\pi$ or $25.13$

**b.** one-fourth of a circle




$\frac{1}{4} \cdot 8\pi = 2\pi$ or $6.28$

**c.** one-third of a circle

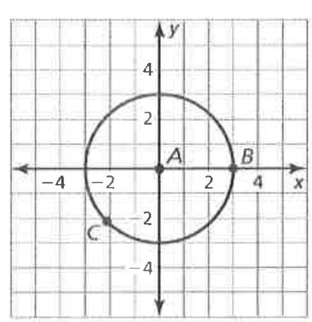



$\frac{1}{3}(8\pi) = \frac{8\pi}{3}$

or

$8.38$

**d.** five-eighths of a circle




$\frac{5}{8} \cdot 8\pi = 5\pi$

or

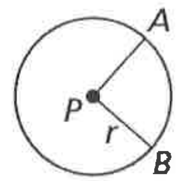
$15.71$

## Arc Length

In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.

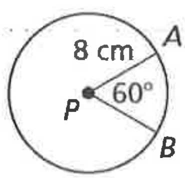
$$\frac{\text{Arc length of } \widehat{AB}}{2\pi r} = \frac{m\widehat{AB}}{360°}, \text{ or}$$

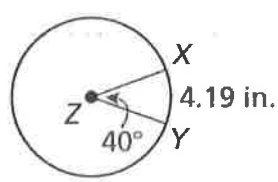
$$\text{Arc length of } \widehat{AB} = \frac{m\widehat{AB}}{360°} \cdot 2\pi r$$

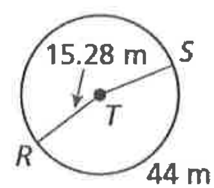



**Find each indicated measure.** ~~**Leave answer in terms of π (exact)**~~ ~~**and**~~ **rounded to the nearest hundredth.**

**a.** arc length of $\widehat{AB}$ **b.** circumference of $\odot Z$ **c.** $m\widehat{RS}$




a.

$\frac{60}{360} \cdot 2\pi(8)$

$\frac{1}{6} \cdot 16\pi = \frac{8\pi}{3}$ cm

$\boxed{= 8.38 \text{ cm}}$

b.

$\frac{4.19}{2\pi r} = \frac{\cancel{40}\ 1}{\cancel{360}\ 9}$

$\frac{2\pi r}{2\pi} = \frac{37.71}{2\pi}$

$r = 59.23$ in

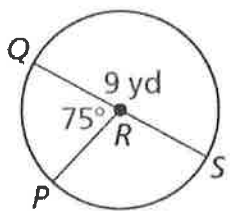
c.

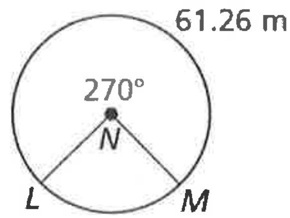
$\frac{44}{2\pi(15.28)} = \frac{m\widehat{RS}}{360}$

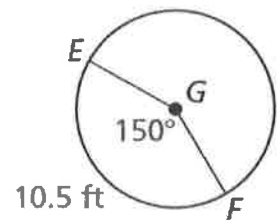
$m\widehat{RS} = 164.99$ m

**Find each indicated measure. Leave answer in terms of π (exact) and rounded to the nearest hundredth.**

**3.** arc length of $\widehat{PQ}$ **4.** circumference of ⊙$N$ **5.** radius of ⊙$G$




$\frac{m\widehat{PQ}}{2\pi(4.5)} = \frac{75}{360}$

$\frac{m\widehat{PQ}}{9\pi} = \frac{5}{24}$

$m\widehat{PQ} = \frac{45\pi}{24} = \frac{15\pi}{8}$

$\boxed{m\widehat{PQ} = 5.89 \text{ yd}}$

$\frac{61.26}{C} = \frac{\cancel{270}\ 3}{\cancel{360}\ 4}$

$3C = 245.04$

$\boxed{C = 81.68 \text{ m}}$

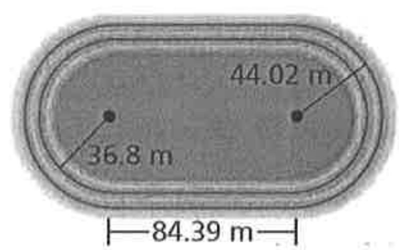
$\frac{10.5}{2\pi r} = \frac{\cancel{150}\ 5}{\cancel{360}\ 12}$

$10\pi r = 126$

$r = \frac{126}{10\pi} = \frac{63\pi}{5}$

$\boxed{r = 39.58 \text{ ft}}$

The curves at the ends of the track shown are 180° arcs of circles. The radius of the arc for a runner on the red path shown is 36.8 meters. About how far does this runner travel to go once around the track?




red path: inner
blue path: outter

The radius of the arc for the blue path is 44.02 m.

How far does this runner travel? Round to the nearest tenth of a meter.

red

$C = 2\pi(36.8)$

$C = 73.6\pi$

$73.6\pi + 2(84.39)$

400.00 m

blue

$C = 2\pi(44.02)$

$C = 88.04\pi$

$88.04\pi + 2(84.39)$

445.37 m

~~In Example 4, the radius of the arc for a runner on the blue path is 44.02 meters, as shown in the diagram. About how far does this runner travel to go once around the track? Round to the nearest tenth of a meter.~~

## Converting between Degrees and Radians

**Degrees to radians**

Multiply degree measure by

$$\frac{2\pi \text{ radians}}{360^\circ}, \text{ or } \frac{\pi \text{ radians}}{180^\circ}.$$

**Radians to degrees**

Multiply radian measure by

$$\frac{360^\circ}{2\pi \text{ radians}}, \text{ or } \frac{180^\circ}{\pi \text{ radians}}.$$

Video: What is a radian?

**a.** Convert 45° to radians.

$$45^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{4} \text{ radians}$$

**b.** Convert $\frac{3\pi}{2}$ radians to degrees.

$$\frac{3\cancel{\pi}}{\cancel{2}} \cdot \frac{\cancel{180}}{\cancel{\pi}} = 270^\circ$$

**c.** Convert 15° to radians.

$$15^\circ \cdot \frac{\pi}{180} = \frac{\pi}{12} \text{ radians}$$

d. Convert $\frac{4\pi}{3}$ radians to degrees.

$$\frac{4\cancel{\pi}}{3} \cdot \frac{\overset{60}{\cancel{180}}}{\cancel{\pi}} = 240^\circ$$

# Homework

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