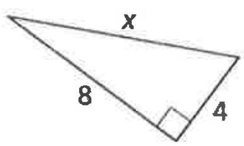
11.3A - Areas of Polygons

Find the value of *x* in the right triangle.

1.



$4^2 + 8^2 = x^2$

$16 + 64 = x^2$

$\sqrt{80} = \sqrt{x^2}$

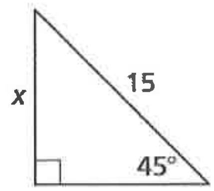
$\sqrt{16}\sqrt{5}$

$\sqrt{4}\sqrt{4}$

$\boxed{4\sqrt{5} = x}$

$x \approx 8.94$

2.



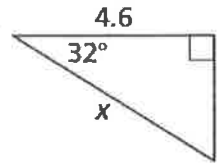
$15 = \ell\sqrt{2}$

$\ell = \frac{15}{\sqrt{2}} = \frac{15\sqrt{2}}{2}$

$\boxed{x = \frac{15\sqrt{2}}{2}}$

$x \approx 10.61$

3.



$\frac{\cos 32}{1} = \frac{4.6}{x}$

$x = \frac{4.6}{\cos 32}$

$\boxed{x = 5.42}$

Determine if the statement is always, sometimes, or never true.

1. Isosceles triangles are similar. sometimes

2. The sum of the lengths of two sides of a triangle is greater than the length of the third side. always

3. A square is a rhombus. always

4. Opposites sides of a kite are parallel. never

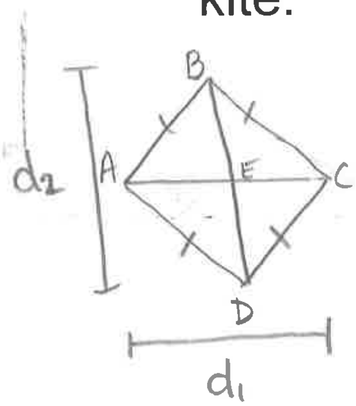
5. The diagonals of a parallelogram bisect each other. always

6. An equilateral polygon is regular. sometimes (rhombus)

Area of a Rhombus or Kite

The area of a rhombus or kite with diagonals d_1 and d_2 is $\frac{1}{2}d_1 d_2$.

Deriving formulas for area of a rhombus or kite:

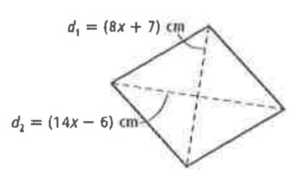


$$A_{\triangle ABC} = \frac{1}{2}bh$$
$$= \frac{1}{2}(d_1)\left(\frac{1}{2}d_2\right)$$
$$= \frac{1}{4}d_1 d_2$$

$\triangle ABC \cong \triangle ADC$ by SSS

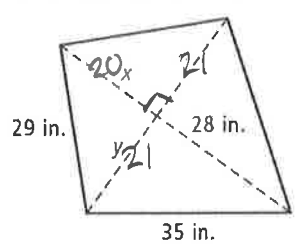
$$A_{ABCD} = 2\left(\frac{1}{4}d_1 d_2\right) = \frac{1}{2}d_1 d_2$$

Find the area of the rhombus.



$$A = \frac{1}{2}(8x+7)(14x-6)$$
$$A = \frac{1}{2}(112x^2 - 48x + 98x - 42)$$
$$A = \frac{1}{2}(112x^2 + 50x - 42)$$
$$\boxed{A = (56x^2 + 25x - 21)\text{ cm}}$$

Find the area of the kite.



$y = 3(7) = 21$ in (3-4-5 triplet)

$$21^2 + x^2 = 29^2$$
$$441 + x^2 = 841$$
$$x^2 = 400$$
$$x = 20$$

$$A = \frac{1}{2}(20+28)(21+21)$$
$$A = \frac{1}{2}(48)(42)$$
$$\boxed{A = 1008 \text{ in}^2}$$

Find d_2 of a rhombus in which d_1 = 3x meters and A = 12xy meters².

$$A = \frac{1}{2}d_1 d_2$$
$$12xy = \frac{1}{2}(3x)d_2$$
$$\frac{24xy}{3x} = \frac{3x d_2}{3x}$$
$$\boxed{8y = d_2}$$

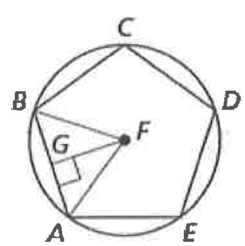
Regular Polygon - All angles and sides ≅

The **center of a regular polygon** is equidistant from the vertices.
The **apothem** is the distance from the center to a side.

A **central angle of a regular polygon** has its vertex at the center, and its sides pass through consecutive vertices.

Each central angle measure of a regular *n*-gon is $\frac{360°}{n}$.

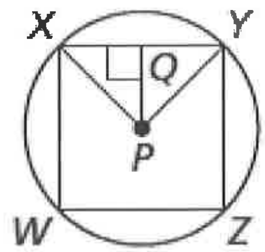
In the diagram, *ABCDE* is a regular pentagon inscribed in $\odot F$. Find each angle measure.



a. $m\angle AFB$ **b.** $m\angle AFG$ **c.** $m\angle GAF$

$m\angle AFB = \frac{360}{5} = 72°$ $m\angle AFG = \frac{72}{2} = 36°$ $m\angle GAF = 90 - 36 = 54°$

In the diagram, *WXYZ* is a square inscribed in ⊙*P*.



3. Identify the center, a radius, an apothem, and a central angle of the polygon.

center P   Apothem $\overline{PQ}$
radius $\overline{PX}$ or $\overline{PY}$ Central Angle $\angle XPY$

4. Find $m\angle XPY$, $m\angle XPQ$, and $m\angle PXQ$.

$m\angle XPY = \frac{360}{4} = 90°$

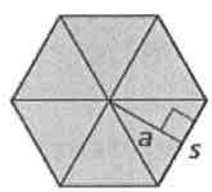
$m\angle XPQ = \frac{90}{2} = 45°$

$m\angle PXQ = 45°$

Area of a Regular Polygon

The area of a regular n-gon with side length s is one-half the product of the apothem a and the perimeter P.

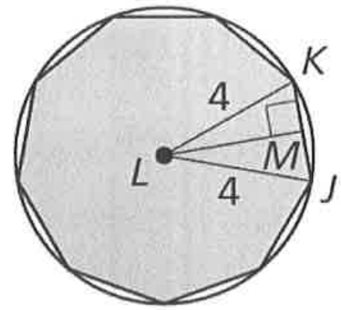
$$A = \frac{1}{2}aP, \text{ or } A = \frac{1}{2}a \cdot ns$$



A regular nonagon is inscribed in a circle with a radius of 4 units. Find the area of the nonagon. Round to nearest hundredth.

$A = \frac{1}{2}ans$

$\boxed{A = 12.50 \text{ units}^2}$



$a = 4\cos 20$

$n = 9$

$s = 8\sin 20$

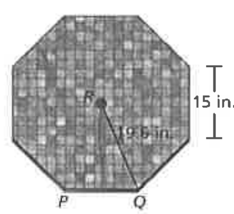
$m\angle L = \frac{360}{9} = 40° \rightarrow m\angle KLM = 20°$

$\frac{\cos 20}{1} = \frac{a}{4}$ $\qquad$ $\frac{\sin 20}{1} = \frac{MK}{4}$

$a = 4\cos 20$ $\qquad$ $MK = 4\sin 20$

$s = 2(4\sin 20)$ or $8\sin 20$

You are decorating the top of a table by covering it with small ceramic tiles. The tabletop is a regular octagon with 15-inch sides and a radius of about 19.6 inches. What is the area you are covering?



$A = \frac{1}{2}ans$

$\boxed{A = 1086.48 \text{ in}^2}$

$a = 19.6\cos 22.5$

$n = 8$

$s = 15$

$m\angle R = \frac{360}{8} = 45$

Half $= 22.5°$

$\frac{\cos 22.5}{1} = \frac{a}{19.6}$

$a = 19.6\cos 22.5$

How many sq. feet is that?

$\frac{1086.48}{(12)^2} = \boxed{7.55 \text{ ft}^2}$

7 sides

Find the area of regular heptagon with side length 2 ft to the nearest tenth.

$\boxed{A = 14.5\ ft^2}$

$A = \frac{1}{2}ans$

$a = 1/\tan 25.715$

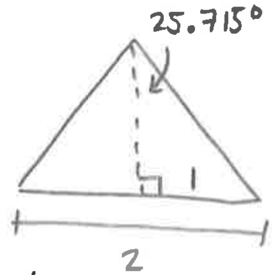
$n = 7$

$s = 2$

$m\angle C = \frac{360}{7} = 51.43$

Half: 25.715

$\frac{\tan 25.715}{1} = \frac{1}{a}$



Find the area of a regular ~~dodecagon~~ polygon with ~~side~~ with perimeter of 60 side length 5 cm to the nearest tenth.

$\boxed{A = 279.9\ cm^2}$

$A = \frac{1}{2}ans$

$a = 2.5/\tan 15$

$n = 12$

$s = 5$

$m\angle C = \frac{360}{12} = 30$

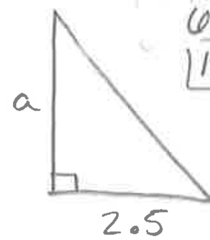
Half: 15°

$\frac{\tan 15}{1} = \frac{2.5}{a}$

$P = ns$

$60 = n(5)$

$\boxed{12 = n}$



* **Find the area of a regular ~~octagon~~ polygon with ~~a side length of 4 cm.~~** an apothem of 4 cm and central angle of 45°

$\boxed{A = 53.0\ cm^2}$

$A = \frac{1}{2}ans$

$a = 4$

$n = 8$

$s = 8\tan 22.5$

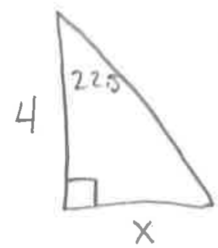
$m\angle C = 45$

$\frac{360}{n} = 45 \rightarrow \frac{45n}{45} = \frac{360}{45}$

$n = 8$

Half = 22.5°

Regular Polygon Generator



$\frac{\tan 22.5}{1} = \frac{x}{4}$

$x = 4\tan 22.5$

So, $S = 2(4\tan 22.5)$

or

$8\tan 22.5$

Connecting Area of a Regular Polygon to the Area of a Circle:

• **Exit Ticket:** Find the area of a stop sign when the side length is 10 inches.

Homework:

pg. 614 #4-12, 15-24

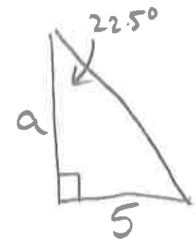
$A = \frac{1}{2} ans$

$a = 5/\tan 22.5$

$n = 8$

$S = 10$

$\boxed{A = 482.8 \text{ in}^2}$



$m\angle C = \frac{360}{8} = 45$

Half: $\frac{45}{2} = 22.5°$

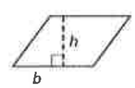
$\frac{\tan 22.5}{1} = \frac{5}{a}$

$a = \frac{5}{\tan 22.5}$

11.3B - Area of Polygons and Composite Figures

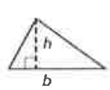
Area **Parallelogram**

The area of a parallelogram with base b and height h is $A = bh$.

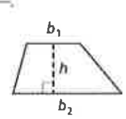


Area **Triangles and Trapezoids**

The area of a triangle with base b and height h is $A = \frac{1}{2}bh$.



The area of a trapezoid with bases b_1 and b_2 and height h is $A = \frac{1}{2}(b_1 + b_2)h$, or $A = \frac{(b_1 + b_2)h}{2}$.



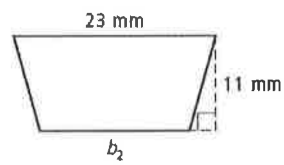
Find the base of the triangle, in which $A = (15x^2)$ cm^2 and height = $10x$ cm ~~cm².~~

$A = \frac{1}{2}bh$

$15x^2 = \frac{1}{2}(b)(10x)$

$15x^2 = b(5x)$ → $\boxed{b = 3x \text{ cm}}$

Find b_2 of the trapezoid, in which $A = 231$ mm^2.



$A = \frac{1}{2}(b_1 + b_2)h$

$231 = \frac{1}{2}(23 + b_2)(11)$

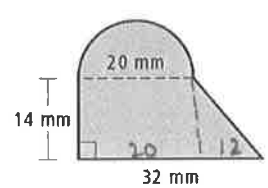
$462 = (23 + b_2)(11)$

$42 = 23 + b_2$

$\boxed{19 = b_2}$

A **composite figure** is made up of simple shapes, such as triangles, rectangles, trapezoids, and circles.

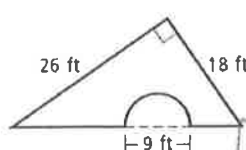
Find the shaded area. Leave your answer in terms of π and round to the nearest hundredth.



$A_{semicircle} = \frac{1}{2}\pi(10)^2 = \frac{100\pi}{2} = 50\pi$

$A_{TRAP} = \frac{1}{2}(20 + 32)(14) = \frac{1}{2}(52)(14) = 364$

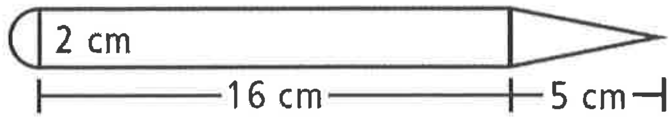
$\boxed{A_{TOTAL} = 50\pi + 364 \approx 521.08 \text{ mm}^2}$



$A_{\triangle} = \frac{1}{2}(18)(26) = 234$

$A_{semicircle} = \frac{1}{2}\pi\left(\frac{9}{2}\right)^2 = \frac{1}{2} \cdot \frac{81\pi}{4} = \frac{81\pi}{8}$

$\boxed{A_{TOTAL} = 234 - \frac{81\pi}{8} \approx 202.19 \text{ ft}^2}$

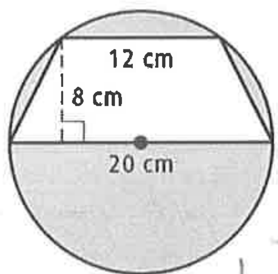


$A_{semicircle} = \frac{1}{2}\pi(1)^2 = \frac{\pi}{2}$

$A_{RECT} = 2(16) = 32$

$A_{\triangle} = \frac{1}{2}(2)(5) = 5$

$$\boxed{A_{TOTAL} = \left(\frac{\pi}{2} + 37\right) cm^2 \text{ or } 38.57\ cm^2}$$



$A_{circle} = \pi(10)^2 = 100\pi$

$A_{TRAP} = \frac{1}{2}(20+12)(8) = 128$

$$\boxed{A_{SHADED} = (100\pi - 128)\ cm^2 \text{ or } 186.16\ cm^2}$$

WS 11.3B