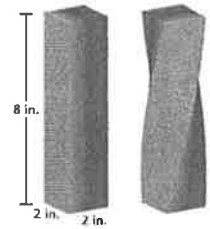
11.5 Volume of Prisms and Cylinders

The volume of a solid is the number of cubic units contained in its interior.

Cavalieri's Principle states: if two soids have the same height and cross-sectional at every level, then they have the same volume.



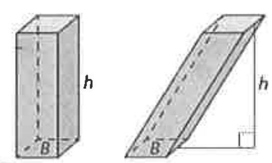
Core Concept

Volume of a Prism

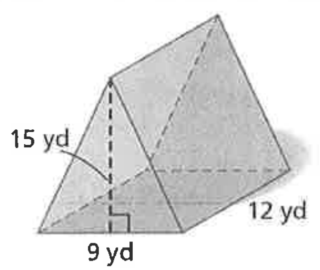
The volume V of a prism is

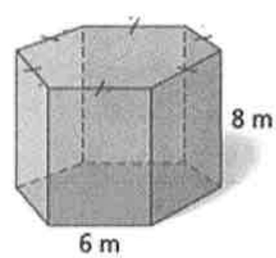
$$V = Bh$$

where B is the area of a base and h is the height.



Find the volume of each prism.





$V = Bh$

$V = \frac{1}{2}(9)(15) \cdot 12$

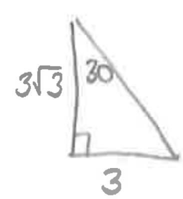
$\boxed{V = 810 \text{ yd}^3}$

$B = \frac{1}{2} ans$

$a = 3\sqrt{3}$

$n = 6$

$s = 6$



$m\angle C = \frac{360}{6} = 60^\circ$

$\frac{1}{2} m\angle C = 30^\circ$

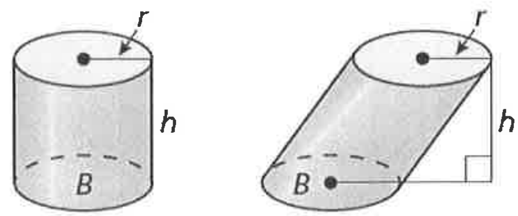
$V = \frac{1}{2}(3\sqrt{3})(6)(6) \cdot 8 = \boxed{432\sqrt{3} \text{ m}^3 \quad 748.25 \text{ m}^3}$

Volume of a Cylinder

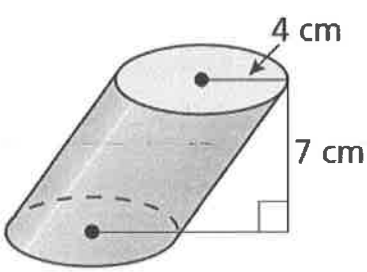
The volume V of a cylinder is

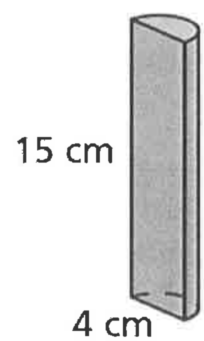
$$V = Bh = \pi r^2 h$$

where B is the area of a base, h is the height, and r is the radius of a base.



Find the exact volume of each cylinder.





$V = \pi r^2 \cdot h$

$V = \pi(4)^2 \cdot 7$

$V = 112\pi \text{ cm}^3$

or

351.86 cm^3

$V = \dfrac{\pi r^2 \cdot h}{2}$

$V = \dfrac{\pi (2)^2 \cdot 15}{2}$

$V = 30\pi \text{ cm}^3$

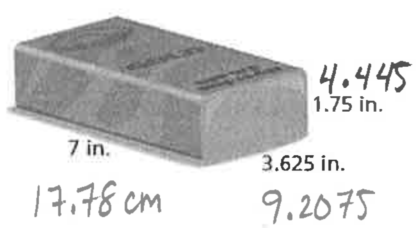
or

94.25 cm^3

<u>Density</u> - the amount of matter than an object has in a given unit of volume.

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

The diagram shows the dimensions of a standard gold bar at Fort Knox. Gold has a density of 19.3 grams per cubic centimeter. Find the mass of a standard gold bar to the nearest gram. (1 inch = 2.54 cm)



$V = (17.78)(9.2075)(4.445)$

$V = 727.69$

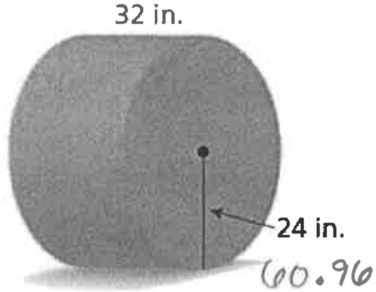
$$19.3\ g/cm^3 = \frac{m}{727.69}$$

$\boxed{14{,}044.38\,g = m}$

$d = \frac{m}{V}$

3. The diagram shows the dimensions of a concrete cylinder. Concrete has a density of 2.3 grams per cubic centimeter. Find the mass of the concrete cylinder to the nearest gram. (1 in = 2.54 cm)

81.28



60.96

$d = \frac{m}{V}$

$V = \pi r^2 \cdot h$

$V = \pi(60.96)^2 \cdot 32$

$V = 373585.2902$

$\frac{2.3 \text{ g/cm}^3}{1} = \frac{m}{373585.2902}$

$859{,}246 \text{ g} = m$

Food The world's largest ice cream cake, built in New York City on May 25, 2004, was approximately a 19 ft by 9 ft by 2 ft rectangular prism. Estimate the volume of the ice cream cake in gallons. If the density of the ice cream was 4.73 pounds per gallon, estimate the weight of the cake. (*Hint:* 1 gallon ≈ 0.134 cubic feet)

$V = 19(19)(2)$

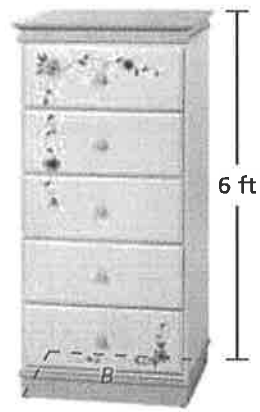
$V = 722 \text{ ft}^3$

$\boxed{722 \text{ ft}^3 \cdot \frac{1 \text{ gal}}{0.134 \text{ ft}^3} = 5388.06 \text{ gallons}}$

weight = (5388.06 gal)(4.73 lbs/gal)

$\boxed{\text{weight} = 25{,}485.52 \text{ lbs}}$

You are building a 6-foot-tall dresser. You want the volume to be 36 cubic feet. What should the area of the base be? Give a possible length and width.



$V = 36\text{ ft}^3$

$V = B \cdot h$

$36 = B(6)$

$\boxed{6\text{ ft}^2 = B}$

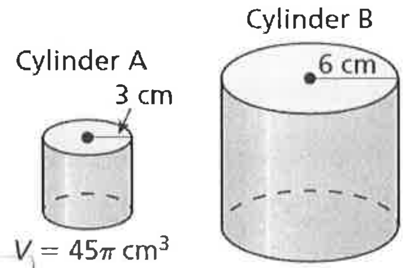
Similar Solids

Two solids of the same type with equal ratios of corresponding linear measures, such as heights or radii, are called similar solids. The ratio of the corresponding linear measures of two similar solids is called the *scale factor*. If two similar solids have a scale factor of k, then the ratio of their volumes is equal to k^3.

$K = 2$

Cylinder A and cylinder B are similar.

Find the volume of cylinder B.

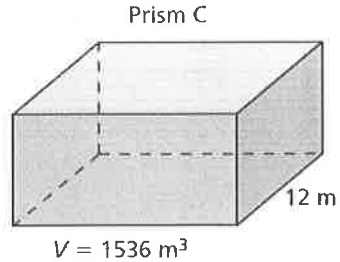


$V_B = V_A \cdot k^3$

$= 45\pi \cdot (2)^3$

$\boxed{V = 360\pi \text{ cm}^3}$

Prism C and prism D are similar. Find the volume of prism D.



$K = \frac{3}{12} = \frac{1}{4}$

$V_D = V_C \cdot k^3$

$V_D = 1536 \left(\frac{1}{4}\right)^3$

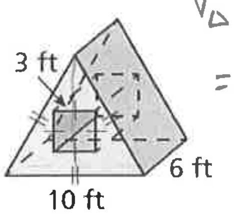
$= 1536 \cdot \frac{1}{64}$

$\boxed{V_D = 24 \text{ m}^3}$

Find the volume of the composite solid.

height of $\triangle$

30, 60, $5\sqrt{3}$, 5



$V_{\triangle prism} = B \cdot h$

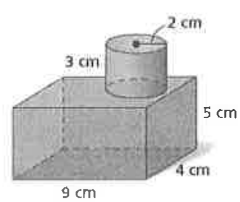
$= \frac{1}{2}(5\sqrt{3})(10) \cdot 6$

$= 150\sqrt{3}$

$V_{RECT} = Bh$

$= (3)(3)(6) = 54$

$\boxed{V_{TOTAL} = (150\sqrt{3} - 54) \text{ ft}^3 \approx 205.81 \text{ ft}^3}$



$V_{CYL} = Bh = \pi r^2 \cdot h$

$= \pi(2)^2 3 = 12\pi$

$V_{Prism} = Bh$

$= (9)(4)(5)$

$= 180$

$\boxed{V_{TOTAL} = (180 + 12\pi) \text{ cm}^3 \approx 217.70 \text{ cm}^3}$

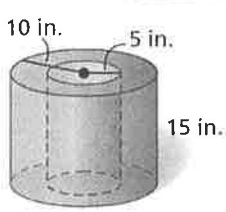
$V_{CYL1} = \pi r^2 \cdot h$

$= \pi(10)^2 \cdot 15$

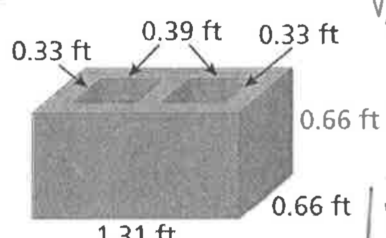
$= 1500\pi$

$V_{CYL2} = \pi(5)^2 \cdot 15$

$= 375\pi$



$\boxed{V_{TOTAL} = 1500\pi - 375\pi = 1125\pi \text{ in}^3 \approx 3534.29 \text{ in}^3}$



$V_{BLOCK} = B \cdot h$

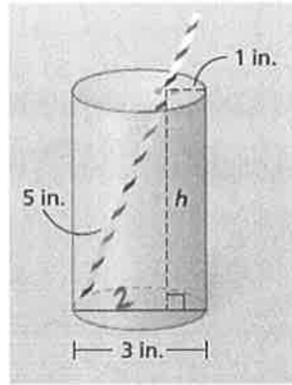
$= .570636$

$V_{SPACE} = .084942$

$\boxed{V_{TOTAL} = .401 \text{ ft}^3}$

A cylindrical juice container with a 3 in. diameter has a hole for a straw that is 1 in. from the side. Up to 5 in. of a straw can be inserted.

a. Find the height h of the container to the nearest tenth.
b. Find the volume of the container to the nearest tenth.
c. How many ounces of juice does the container hold? (*Hint:* $1 \text{ in}^3 \approx 0.55$ oz)



a) $2^2 + h^2 = 5^2$
$4 + h^2 = 25$
$h^2 = 21$
$h = \sqrt{21} \approx 4.6 \text{ in}^3$

b) $V = \pi r^2 \cdot h$
$= \pi \left(\frac{3}{2}\right)^2 \cdot \sqrt{21}$
$= \frac{9\pi\sqrt{21}}{4}$
$\approx 32.4 \text{ in}^3$

c) $32.4 \text{ in}^3 \cdot \frac{.55 \text{ oz}}{1 \text{ in}^3} = 17.82$ oz

Homework

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