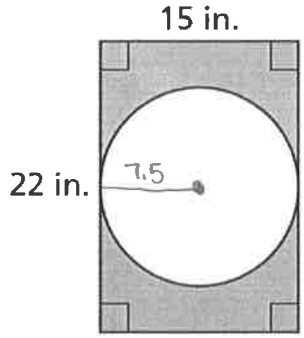
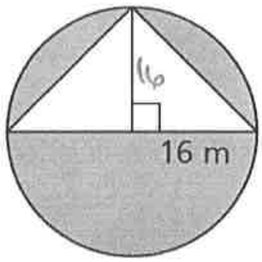
# 11.7 Surface Area and Volume of Cones

**Find the area of the shaded region.**

**2.**




**3.**




$A_{\square} = 22 \cdot 15$ $A_{\circ} = \pi(7.5)^2$

$= 330$ $= 56.25\pi$

$\boxed{330 - 56.25\pi \text{ in}^2}$

$\boxed{153.3 \text{ in}^2}$

$A_{\circ} = \pi(16)^2 - A_{\Delta} = \frac{1}{2}(16)(16)$

$= 256\pi$ $= 128$

$\boxed{256\pi - 128 \text{ m}^2}$

$\boxed{676.2 \text{ m}^2}$

**Find an equation of the line.**

**1.** parallel to the line $y = 2x + 3$, passes through the point $(0, -8)$

$m = 2$

$-8 = 2(0) + b$

$-8 = b$

$\boxed{y = 2x - 8}$

**2.** perpendicular to the line $y = -5x - 7$, passes through the point $(-1, -3)$

$\perp m = \frac{1}{5}$

$-3 = \frac{1}{5}(-1) + b$

$-3 = -\frac{1}{5} + b$

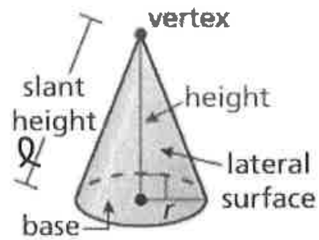
$+\frac{1}{5} \quad +\frac{1}{5}$

$-2\frac{4}{5} = b$

$\boxed{y = \frac{1}{5}x - 2\frac{4}{5}}$

## Finding Surface Areas of Right Cones

Recall that a *circular cone*, or *cone*, has a circular *base* and a *vertex* that is not in the same plane as the base. The *altitude*, or *height*, is the perpendicular distance between the vertex and the base. In a *right cone*, the height meets the base at its center and the *slant height* is the distance between the vertex and a point on the base edge.



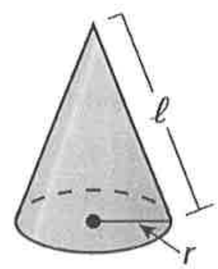

The **lateral surface of a cone** consists of all segments that connect the vertex with points on the base edge. When you cut along the slant height and lay the right cone flat, you get the net shown at the left. In the net, the circular base has an area of $\pi r^2$ and the lateral surface is a sector of a circle. You can find the area of this sector by using a proportion, as shown below.

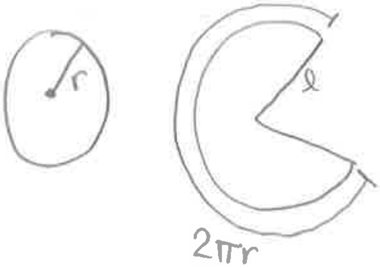
### Surface Area of a Right Cone

The surface area $S$ of a right cone is

$$S = \pi r^2 + \pi r\ell$$

where $r$ is the radius of the base and $\ell$ is the slant height.




$$\frac{\text{Area of Sector}}{\text{Area of Circle}} = \frac{\text{Arc Length}}{\text{Circumference}}$$

$$\frac{x}{\pi\ell^2} = \frac{2\pi r}{2\pi\ell}$$

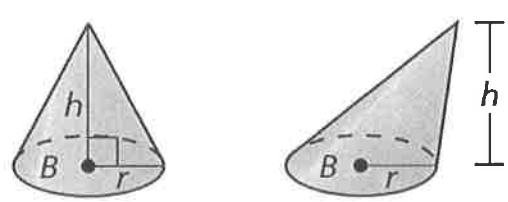
$$\frac{\cancel{2\pi\ell}x}{\cancel{2\pi\ell}} = \frac{\cancel{2\pi}r\cdot\pi\ell^{\cancel{2}}}{\cancel{2\pi\ell}} \quad \boxed{x = \pi r\ell}$$

### Volume of a Cone

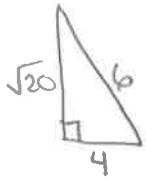
The volume $V$ of a cone is

$$V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$$

where $B$ is the area of a base, $h$ is the height, and $r$ is the radius of the base.

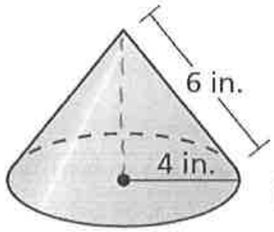


Find the surface area and volume of the right cone.




$4^2 + x^2 = 6^2$

$\sqrt{x^2} = \sqrt{20}$




$S = \pi r^2 + \pi r l$

$= \pi(4)^2 + \pi(4)(6)$

$= 16\pi + 24\pi$

$\boxed{S = 40\pi \text{ in}^2}$

$\boxed{125.7 \text{ in}^2}$

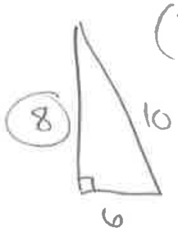
$V = \frac{1}{3}\pi r^2 h$

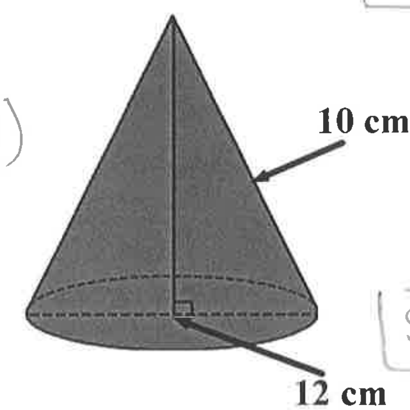
$= \frac{1}{3}\pi(4)^2(\sqrt{20})$

$= 23.9\pi$

$\boxed{V = 74.9 \text{ in}^3}$

(3-4-5)




$S = \pi r^2 + \pi r l$

$= \pi(6)^2 + \pi(6)(10)$

$= 36\pi + 60\pi$

$\boxed{S = 96\pi \text{ in}^2}$

$\boxed{301.6 \text{ in}^2}$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(6)^2(8)$

$\boxed{V = 96\pi \text{ in}^3}$

$\boxed{301.6 \text{ in}^3}$

$D = 12$

$r = 6$

scale factor $k$

$\frac{\text{radius B}}{\text{radius A}} = k$

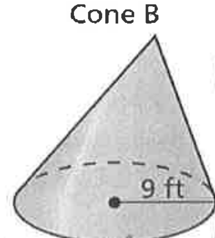
Cone A and cone B are similar.

Find the volume of cone B.

Cone A


$V = 15\pi \text{ ft}^3$

Cone B




$\frac{r_B}{r_A} = \frac{9}{3} = 3$

$k = 3$

$\frac{V_B}{V_A} = k^3 \rightarrow \frac{V_B}{15\pi} = \frac{3^3}{1^3}$

$\frac{V_B}{15\pi} = \frac{27}{1}$

$\boxed{V_B = 405\pi \text{ ft}^3}$

$\boxed{1272.3 \text{ ft}^3}$

Cone C and cone D are similar. Find the volume of cone D.

$\frac{h_D}{h_C} = \frac{2}{8} = \frac{1}{4}$

$k = \frac{1}{4}$

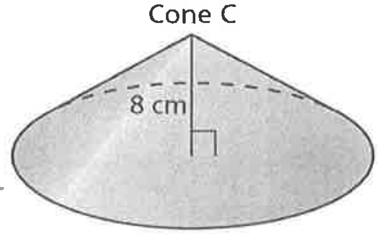
$\frac{V_D}{V_C} = \left(\frac{1}{4}\right)^3$

$\frac{V_D}{384\pi} = \frac{1}{64}$

$\frac{64 V_D}{64} = \frac{384\pi}{64}$

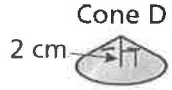
$\boxed{V_D = 6\pi \text{ cm}^3}$

Cone C

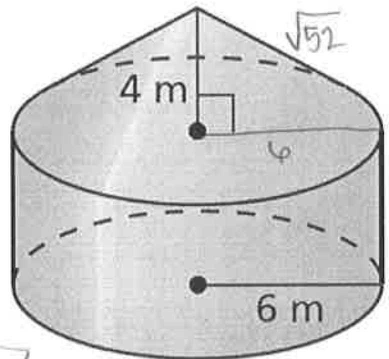


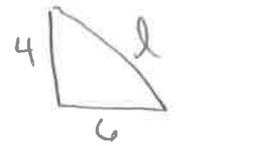

$V = 384\pi \text{ cm}^3$

Cone D



Find the surface area and volume of the composite solid.



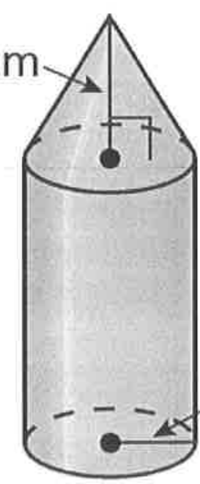



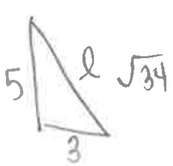

$4^2 + 6^2 = \ell^2$
$\sqrt{52} = \sqrt{\ell^2}$

$S_{solid} = S_{cylinder} + S_{cone} - 2\pi r^2$
$= 2\pi rh + 2\pi r^2 + (\pi r^2 + \pi r\ell) - 2\pi r^2$
$= 2\pi(6)(5) + 2\pi(6)^2 + (\pi(6)^2 + \pi(6)(\sqrt{52})) - 2\pi(6)^2$
$= 60\pi + \cancel{72\pi} + 36\pi + 6\sqrt{52}\pi - \cancel{72\pi}$
$\boxed{S = 96\pi + 6\sqrt{52}\pi \text{ m}^2}$ $\boxed{437.5 \text{ m}^2}$

$V_{solid} = V_{cylinder} + V_{cone}$
$= \pi r^2 h + \frac{1}{3}\pi r^2 h$
$= \pi(6)^2(5) + \frac{1}{3}\pi(6)^2(4)$
$= 180\pi + 48\pi$
$\boxed{V = 228\pi \text{ m}^3}$ $\boxed{716.3 \text{ m}^3}$

**5.** Find the surface area and volume of the composite solid.




$5^2 + 3^2 = \ell^2$
$\sqrt{34} = \sqrt{\ell^2}$

$S = S_{cylinder} + S_{cone} - 2\pi r^2$
$= (2\pi rh + 2\pi r^2) + (\pi r^2 + \pi r\ell) - 2\pi r^2$
$= 2\pi(3)(10) + \pi(3)^2 + \pi(3)(\sqrt{34})$
$= 60\pi + 9\pi + 3\sqrt{34}\pi$
$\boxed{S = 69\pi + 3\sqrt{34}\pi \text{ cm}^2}$ $\boxed{271.7 \text{ cm}^2}$

$V = V_{cylinder} + V_{cone}$
$= \pi r^2 h + \frac{1}{3}\pi r^2 h$
$= \pi(3)^2(10) + \frac{1}{3}\pi(3)^2(5)$
$= 90\pi + 15\pi$
$\boxed{V = 105\pi \text{ cm}^3}$ $\boxed{329.9 \text{ cm}^3}$

# Homework:

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