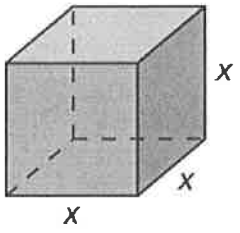
Key

11.8 - Surface Area and Volume of Spheres

Use the diagram and the given surface area to find the value of *x*.

1. $SA = 1350$ in.^{2}



Cube

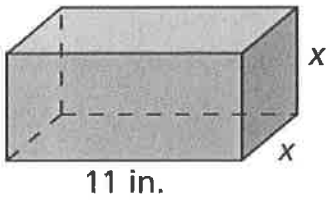
$S = 6s^2$

$\frac{1350}{6} = \frac{6x^2}{6}$

$\sqrt{225} = \sqrt{x^2}$

$15 = x$

2. $SA = 270$ in.^{2}



Rectangular solid

$S = 4(11)(x) + 2x^2$

$270 = 44x + 2x^2$

$-270 \quad -270$

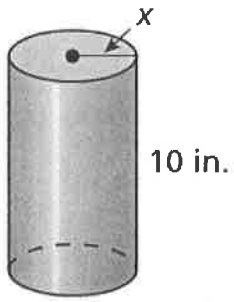
$2x^2 + 44x - 270 = 0$

$2(x^2 + 22x - 135) = 0$

$(x - 5)(x + 27) = 0$

$x = 5, -27$ $\quad x = 5$

3. $SA = 78\pi$ in.^{2}



Cylinder

$S = 2\pi r^2 + 2\pi rh$

$78\pi = 2\pi x^2 + 2\pi x \cdot 10$

$78\pi = 2\pi x^2 + 20\pi x$

$-78\pi \quad -78\pi$

$2\pi x^2 + 20\pi x - 78\pi = 0$

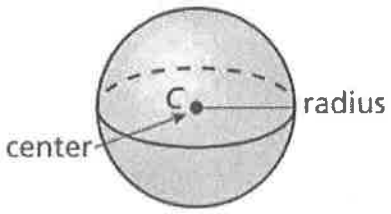
$2\pi(x^2 + 10x - 39) = 0$

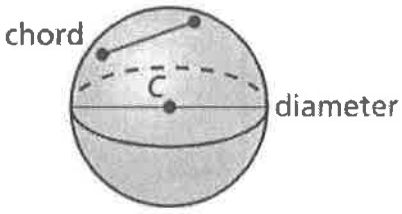
$(x - 3)(x + 13) = 0$

$x = 3, -13$ $\quad x = 3$

Sphere - the set of all points in space equidistant from a given point (called the center)

Radius - segment from center to any point on sphere

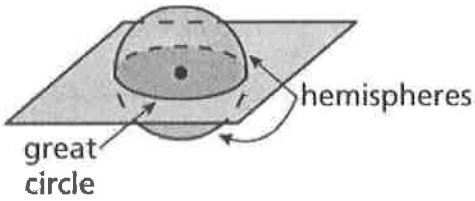




If a plane intersects a sphere, then the intersection is a point or a circle.

If a plane contains the center of the sphere, then the intersection is a great circle of the sphere. The circumference of the great circle is the same as the sphere.

The great circle seperates the sphere into congruent halves called hemispheres.

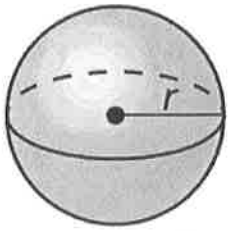


Surface Area of a Sphere

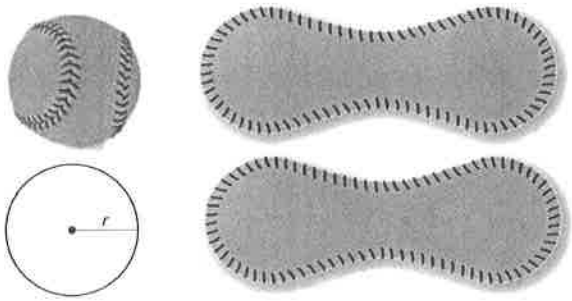
The surface area S of a sphere is

$$S = 4\pi r^2$$

where r is the radius of the sphere.



$S = 4\pi r^2$

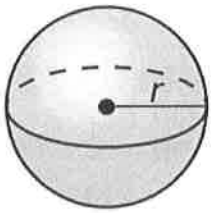


Volume of a Sphere

The volume V of a sphere is

$$V = \frac{4}{3}\pi r^3$$

where r is the radius of the sphere.

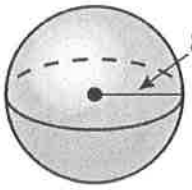


$V = \frac{4}{3}\pi r^3$

Video: Volume of a Sphere

Find the surface area and volume of each sphere.

a.



$S = 4\pi r^2$

$= 4\pi(8)^2$

$S = 256\pi \text{ in}^2$

804.2 in^2

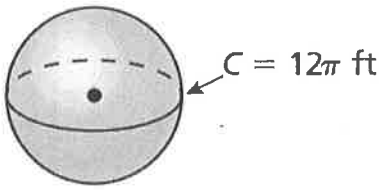
$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(8)^3$

$V = \frac{2048\pi}{3} \text{ in}^3$

2144.7 in^3

b.



$C = 2\pi r$

$\frac{12\pi}{2\pi} = \frac{2\pi r}{2\pi}$

$6 = r$

$S = 4\pi r^2$

$= 4\pi(6)^2$

$S = 144\pi \text{ ft}^2$

452.4 ft^2

$V = \frac{4}{3}\pi r^3$

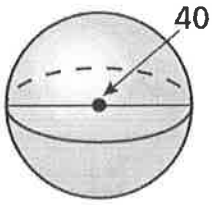
$= \frac{4}{3}\pi(6)^3$

$V = 288\pi \text{ ft}^3$

904.8 ft^3

Find the surface area and volume of each sphere.

1.



$r = 20$

$S = 4\pi r^2$

$= 4\pi(20)^2$

$S = 1600\pi \text{ ft}^2$

5026.5 ft^2

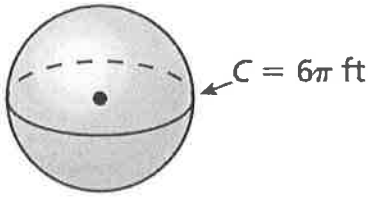
$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(20)^3$

$V = \frac{3200\pi}{3} \text{ ft}^3$

3351.03 ft^3

2.



$C = 2\pi r$

$\frac{6\pi}{2\pi} = \frac{2\pi r}{2\pi}$

$3 = r$

$S = 4\pi r^2$

$= 4\pi(3)^2$

$S = 36\pi \text{ ft}^2$

113.1 ft^2

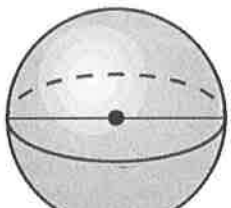
$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(3)^3$

$V = 36\pi \text{ ft}^3$

113.1 ft^3

Find the diameter of the sphere.



$S = 20.25\pi \text{ cm}^2$

$S = 4\pi r^2$

$\frac{20.25\pi}{4\pi} = \frac{4\pi r^2}{4\pi}$

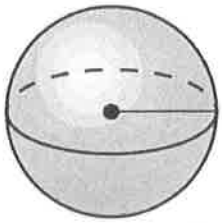
$\sqrt{5.0625} = \sqrt{r^2}$

$2.25 = r$

$D = 2(2.25)$

$D = 4.5 \text{ cm}$

Find the radius of the sphere.



$S = 30\pi \text{ m}^2$

$S = 4\pi r^2$

$\frac{30\pi}{4\pi} = \frac{4\pi r^2}{4\pi}$

$\sqrt{\frac{15}{2}} = \sqrt{r^2}$

$2.74 \text{m} = r$

The surface area of a sphere is 324π square centimeters. Find the volume of the sphere.

$S = 4\pi r^2$

$\frac{324\pi}{4\pi} = \frac{4\pi r^2}{4\pi}$

$\sqrt{81} = \sqrt{r^2}$

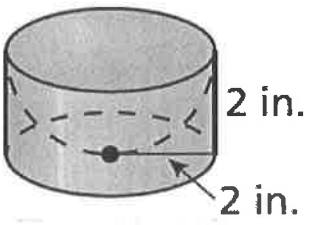
$9 = r$

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi (9)^3$

$V = 972\pi \text{ cm}^3$

Find the volume of the composite solid.



$V_{solid} = V_{cylinder} - V_{hemisphere}$

$= \pi r^2 h - \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$= \pi r^2 h - \frac{2}{3}\pi r^3$

$= \pi (2)^2 (2) - \frac{2}{3}\pi (2)^3$

$= 8\pi - \frac{16}{3}\pi$

$V = \frac{8}{3}\pi \text{ in}^3$

8.38 in^3

The surface area of a sphere is 576π square centimeters. Find the volume of the sphere.

$S = 4\pi r^2$

$\frac{576\pi}{4\pi} = \frac{4\pi r^2}{4\pi}$

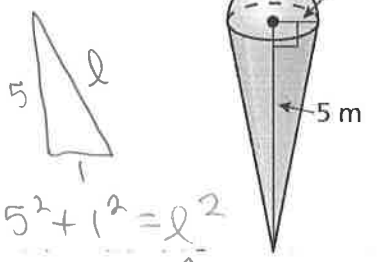
$\sqrt{144} = \sqrt{r^2}$ $r = 12$

$V = \frac{4}{3}\pi(12)^3$

$V = 2304\pi \text{ cm}^3$

7238.2 cm^3

Find the surface area and volume of the composite solid



$5^2 + 1^2 = l^2$

$26 = l^2$

$\sqrt{26} = l$

$S = S_{cone} + S_{hemisphere} - 2\pi r^2$

$= \pi r^2 + \pi r l + \frac{1}{2}(4\pi r^2) - 2\pi r^2$

$= \pi r^2 + \pi r l + 2\pi r^2 - 2\pi r^2$

$= \pi(1)^2 + \pi(1)(\sqrt{26})$

$= \pi + \sqrt{26}\pi$

$S = 19.16 \text{ m}^2$

Homework:

pg. 652 # 4-20 Evens, 23, 24, 28, 36, 37

$V = V_{cone} + V_{hemisphere}$

$= \frac{1}{3}\pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi(1)^2(5) + \frac{2}{3}\pi(1)^3$

$= \frac{5}{3}\pi + \frac{2}{3}\pi$

$V = \frac{7}{3}\pi \text{ m}^3$

7.33 m^3