

12.2 Independent and Dependent Events

Key

A group of 128 students was asked to select their favorite high school sport: basketball, football, lacrosse, or baseball. The table shows the results. Use the results to find the probabilities that a student chosen at random from this group would prefer the following.

Survey Results			
basketball	football	lacrosse	baseball
48	35	20	25

1. lacrosse

$$\frac{20}{128} = \frac{5}{32} \quad 0.156 \quad 15.6\%$$

2. football

$$\frac{35}{128} = 0.27 \quad 27\%$$

3. baseball or basketball

$$\frac{25 + 48}{128} = \frac{73}{128} \quad 0.57 \quad 57\%$$

4. football or lacrosse

$$\frac{35 + 20}{128} = \frac{55}{128} \quad 0.429 \quad 42.9\%$$

5. one of the four sports

$$\frac{128}{128} = 1$$

6. none of the four sports

$$\frac{0}{128} = 0$$

Determine whether the events are independent or dependent. Explain your reasoning.

a. Two six-sided dice are rolled.

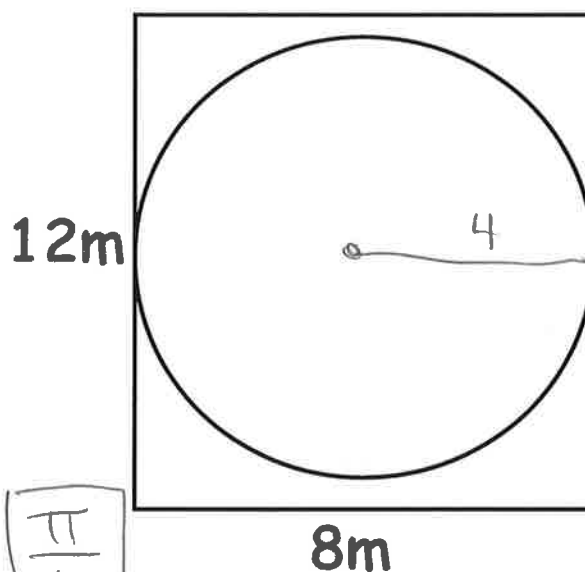
independent

b. Six pieces of paper, numbered 1 through 6, are in a bag. Two pieces of paper are selected one at a time without replacement.

dependent

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If a point inside the 12x8 rectangle is picked at random, what is the probability it will be inside the circle?



$$\frac{\pi(4)^2}{8 \cdot 12} = \frac{16\pi}{96} = \frac{\pi}{6}$$

$$0.524$$

$$52.4\%$$

Remember this?

Adding ("or") 1 draw	no overlap $P(\text{blue or red})$	overlap $P(\text{adult female})$ $P(a) + P(f) - (a/f)$
Multiply ("and") 2+ draws	w/ replacement (independent)	w/o replacement (dependent)

Core Concept

Probability of Independent Events

Words Two events A and B are independent events if and only if the probability that both events occur is the product of the probabilities of the events.

Symbols $P(A \text{ and } B) = P(A) \cdot P(B)$

Core Concept

Probability of Dependent Events

Words If two events A and B are dependent events, then the probability that both events occur is the product of the probability of the first event and the conditional probability of the second event given the first event.

Symbols $P(A \text{ and } B) = P(A) \cdot P(B|A)$

Example Using the information in Example 2:

$$P(\text{girl first and girl second}) = P(\text{girl first}) \cdot P(\text{girl second} | \text{girl first})$$

$$= \frac{9}{12} \cdot \frac{8}{11} = \frac{2}{3}$$

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A six-sided cube is labeled with the numbers 1, 2, 2, 3, 3, and 3. Four sides are colored red, one side is white, and one side is yellow.

Find the probability of rolling a 2, then a 2.



$$\frac{2}{6} \cdot \frac{2}{6} = \frac{4}{36} = \boxed{\frac{1}{9}}$$

Find the probability of rolling red, then white, then yellow.



$$\frac{4}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{4}{216} = \boxed{\frac{1}{54}}$$

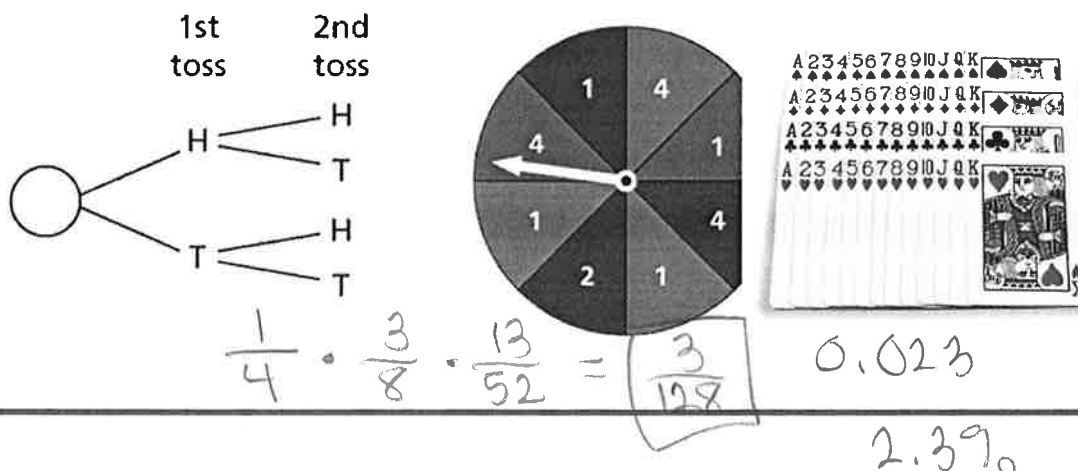
As part of a board game, you need to spin the spinner, which is divided into equal parts. Find the probability that you get a 5 on your first spin and a number greater than 3 on your second spin.



$$\frac{1}{8} \cdot \frac{1}{8} = \boxed{\frac{1}{64}}$$

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If 2 coins are tossed, one spinner is spun, and 1 card is drawn from a deck. What is the probability of 2 heads, a four on the spinner, and the card is a spade?



A bag contains twenty \$1 bills and five \$100 bills.

You randomly draw a bill from the bag, set it aside, and then randomly draw another bill from the bag. Find the probability that both events *A* and *B* will occur.

Event A: The first bill is \$100.

$$\frac{5}{25} = \frac{1}{5} \quad 0.2 \quad 20\%$$

Event B: The second bill is \$100.

$$\frac{5}{25} \cdot \frac{4}{24} = \frac{1}{30} \quad 0.033 \quad 3.39\%$$

12.2 Independent and Dependent Events

You randomly select 3 cards from a standard deck of 52 playing cards. What is the probability that all 3 cards are hearts when (a) you replace each card before selecting the next card, and (b) you do not replace each card before selecting the next card?

Compare the probabilities.

$$a) \frac{13}{52} \cdot \frac{13}{52} \cdot \frac{13}{52} = \boxed{\frac{1}{64}} \quad \begin{array}{l} 0.016 \\ 1.6\% \end{array}$$

$$b) \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} = \boxed{\frac{11}{850}} \quad \begin{array}{l} 0.013 \\ 1.3\% \end{array}$$

3. In Example 3, what is the probability that you spin an even number and then an odd number?

$$\frac{4}{8} \cdot \frac{4}{8} = \boxed{\frac{1}{4}} \quad \begin{array}{l} 0.25 \\ 25\% \end{array}$$

4. In Example 4, what is the probability that both bills are \$1 bills?

$$\frac{20}{25} \cdot \frac{19}{24} = \boxed{\frac{19}{30}} \quad \begin{array}{l} 0.633 \\ 63.3\% \end{array}$$

5. In Example 5, what is the probability that none of the cards drawn are hearts when (a) you replace each card, and (b) you do not replace each card? Compare the probabilities.

$$a) \frac{39}{52} \cdot \frac{38}{51} \cdot \frac{37}{50} = 0.414$$
$$\boxed{41.4\%}$$

12.2 Independent and Dependent Events

A quality-control inspector checks for defective parts. The table shows the results of the inspector's work. Find (a) the probability that a defective part "passes," and (b) the probability that a non-defective part "fails."

a) $\frac{3}{500}$ 0.006

	Pass	Fail
Defective	3	36
Non-defective	450	11

total = 500

b) $\frac{11}{500}$ 0.022

A bag contains 5 red, 3 green, 4 blue, and 8 yellow marbles. Find the probability of randomly selecting a green marble, and then a yellow marble if the first marble is replaced. $5+3+4+8 = 20$

$\frac{3}{20} \cdot \frac{8}{20} = \frac{3}{50}$ 0.06
6%

What if the first marble was not replaced?

$\frac{3}{20} \cdot \frac{8}{19} = \frac{6}{95}$ 0.063
6.3%

12.2 Independent and Dependent Events

At a school, 60% of students buy a school lunch. Only 10% of students buy lunch and dessert. What is the probability that a student who buys lunch also buys dessert?

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

$$= \frac{0.1}{0.6}$$

$$= \boxed{\frac{1}{6}}$$

$$0.167$$

$$16.7\%$$

At a coffee shop, 80% of customers order coffee. Only 15% of customers order coffee and a bagel. What is the probability that a customer who orders coffee also orders a bagel?

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

$$= \frac{0.15}{0.8}$$

$$= \boxed{\frac{3}{16}}$$

$$0.188$$

$$18.8\%$$

Homework

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