2.1A - Conditional Statements

Bellwork

The endpoints of $\overline{CD}$ are given. Find the coordinate specified *and find the length of CD.*

1. $C(4, -6)$ and $D(8, 8)$

Find M and CD.

$\left(\frac{12}{2}, \frac{2}{2}\right) \rightarrow (6,1)$

$d = \sqrt{(8-4)^2 + (14)^2}$

$d = \sqrt{16 + 196}$

$d = \sqrt{212} = 2\sqrt{53}$

212 → 2 · 106 → 2 · 53

2. $M(-1, 1)$ and $D(1, -4)$

Find C and CD.

$(-3, 6)$

$d = \sqrt{(1+3)^2 + (-4-6)^2}$

$d = \sqrt{(4)^2 + (-10)^2} = \sqrt{116}$

116 → 2 · 58 → 2 · 29

$d = 2\sqrt{29}$

Cumulative Warm Up

Complete the statement.

1. A hexagon has six sides.

2. If two lines form a right angle, they are perpendicular.

3. Two angles that form a right angle are complementary angles.

4. A straight angle has measure of 180°.

Warm Up

Work with a partner. Determine whether each conditional statement is true or false. Justify your answer.

a. If yesterday was Wednesday, then today is Thursday.

True

b. If an angle is acute, then it has a measure of 30°.

False, an angle could be acute and be 20°

c. If a month has 30 days, then it is June.

False, April has 30 days

Exploration 1

Work with a partner. Determine whether each conditional statement is true or false. Justify your answer.

a. If △*ADC* is a right triangle, then the Pythagorean Theorem is valid for △*ADC*. True

b. If ∠*A* and ∠*B* are complementary, then the sum of their measures is 180°. False, they will add to 90

c. If figure *ABCD* is a quadrilateral, then the sum of its angle measures is 180°. False, angles will add to 180

d. If points *A*, *B*, and *C* are collinear, then they lie on the same line.

True

e. If $\overleftrightarrow{AB}$ and $\overleftrightarrow{BD}$ intersect at a point, then they form two pairs of vertical angles. True

Exploration 3

Core Concept

Conditional Statement

A **conditional statement** is a logical statement that has two parts, a *hypothesis p* and a *conclusion q*. When a conditional statement is written in **if-then form**, the "if" part contains the **hypothesis** and the "then" part contains the **conclusion**.

Words If p, then q. **Symbols** $p \rightarrow q$ (read as "p implies q")

Core Concept

Label the hypothesis with "p" and the conclusion with "q". Then rewrite the conditional statement in if-then form.

a. All birds have feathers.

If an animal is a bird, then it has feathers.

b. You are in Texas if you are in Houston.

If you are in Houston, then you are in Texas.

Example 1

Label the hypothesis with "p" and the conclusion with "q". Then rewrite the conditional statement in if-then form. **Then rewrite the conditional statement in if-then form.**

1. All 30° angles are acute angles.

p q

If an angle is 30°, then it is acute.

2. $2x + 7 = 1$, because $x = -3$.

q p

If $x=-3$, then $2x+7=1$

Monitoring Progress 1-2

Core Concept

Negation

The **negation** of a statement is the *opposite* of the original statement. To write the negation of a statement p, you write the symbol for negation ($\sim$) before the letter. So, "not p" is written $\sim p$.

Words not p **Symbols** $\sim p$

Core Concept

Write the negation of each statement.

a. The ball is red.

The ball is not red

b. The cat is *not* black.

The cat is black

c. The shirt is green.

The shirt is not green

d. The shoes are *not* red.

The shoes are red.

Example 2

Core Concept

Related Conditionals

Consider the conditional statement below.

Words If p, then q. **Symbols** $p \to q$

Converse To write the **converse** of a conditional statement, exchange the hypothesis and the conclusion.

Words If q, then p. **Symbols** $q \to p$

Inverse To write the **inverse** of a conditional statement, negate both the hypothesis and the conclusion.

Words If not p, then not q. **Symbols** $\sim p \to \sim q$

Contrapositive To write the **contrapositive** of a conditional statement, first write the converse. Then negate both the hypothesis and the conclusion.

Words If not q, then not p. **Symbols** $\sim q \to \sim p$

A conditional statement and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional statement are either both true or both false. In general, when two statements are both true or both false, they are called **equivalent statements**.

Core Concept

Let p be "you are a guitar player" and let q be "you are a musician." Write each statement in words. Then decide whether it is *true* or *false*.

a. the conditional statement $p \rightarrow q$

If you are a guitar player, then you are a musician.

b. the converse $q \rightarrow p$

If you are a musician, then you are a guitar player.

c. the inverse $\sim p \rightarrow \sim q$

If you are not a guitar player, then you are not a musician

d. the contrapositive $\sim q \rightarrow \sim p$

If you are not a musician, then you are not a guitar player.

Example 3

Let p be "the stars are visible" and let q be "it is night."

Write each statement in words. Then decide whether it is *true* or *false*.

a. the conditional statement

If the stars are visibe, then it is night.

b. the converse

If it is night, then the stars are visible.

c. the inverse

If the stars are not visible, then it is not night.

d. the contrapositive

If it is not night, then the stars are not visible.

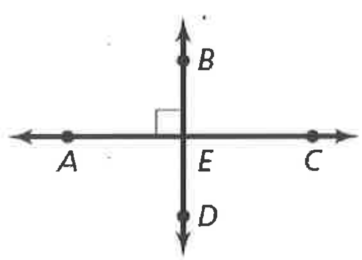
Sep 14-3:54 PM

Definitions: you can write a definition as a conditional statement in if-then form, or as its converse; both are true for definitions.

perpendicular lines: lines that intersect to form 90° angles

Sep 10-2:56 PM

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.



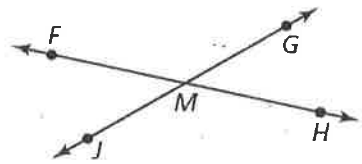
a. $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$ True

b. $\angle AEB$ and $\angle CEB$ are a linear pair. True

c. $\overrightarrow{EA}$ and $\overrightarrow{EB}$ are opposite rays. False

Example 4

Use the diagram. Decide whether the statement is true. Explain your answer using the definitions you have learned.



a. $\angle JMF$ and $\angle FMG$ are supplementary. True

b. Point M is the midpoint of $\overline{FH}$. False

c. $\angle JMF$ and $\angle HMG$ are vertical angles. True

d. $\overleftrightarrow{FH} \perp \overleftrightarrow{JG}$ False

Monitoring Progress 6-9

Homework:

~~pg. 71 # 2, 3, 8, 10, 18, 22, 25-28, 38~~

2-4, 8-10, 14-15, 18-20, 25-28, 38

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2.1B - Biconditional Statements

1. Given the conditional statement, which is always true? (Converse, Inverse, Contrapositive)

"If John lives in Chicago, then he lives in Illinois."

2. Find a counterexample.

If two angles are congruent, then they are acute.

$m\angle A = 100^\circ$, $m\angle B = 100^\circ$

Sep 10-2:58 PM

Core Concept

Biconditional Statement

When a conditional statement and its converse are both true, you can write them as a single *biconditional statement*. A **biconditional statement** is a statement that contains the phrase "if and only if."

Words p if and only if q **Symbols** $p \leftrightarrow q$

Any definition can be written as a biconditional statement.

***A biconditional statement must be true both ways!

Core Concept

Rewrite the following definitions as biconditional statements.

1. If two lines intersect to form a right angle, then they are perpendicular lines. Two lines intersect to form a right ∠ iff they are perpendicular lines.

2. A right angle has a measure of 90°.

An angle is a right angle iff it measures 90°.

3. If two segments have the same length, then they are congruent.

Two segments have the same length iff they are congruent.

Sep 10-3:00 PM

Rewrite the statements as a single biconditional statement.

1. If Mary is in theater class, then she will be in the fall play. If Mary is in the fall play, then she must be taking theater class.

Mary is in theatre class iff she is in the fall play.

2. If you can run for President, then you are at least 35 years old. If you are at least 35 years old, then you can run for President.

You can run for President iff you are at least 35 years old.

Monitoring Progress 12-13

Let *p* be "snow is falling" and let *q* be "it is winter." Write the conditional statement ($p \rightarrow q$), converse, inverse, and contrapositive in words.

If snow is falling, then it is winter.

If it is winter, then snow is falling

If snow is not falling, then it's not winter.

If it is not winter, then snow is not falling.

Could a true biconditional statement be written from these statements?

No

Closure

Homework for 2.1B:

pg. 72 # ~~30-36 Evens~~, ~~45ab~~, 46, ~~53~~, 55, 58, 59

pg 72 # 29-36, 46, 55, 58, 59

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