

2.2 Inductive and Deductive Reasoning

1. A, B, and C are collinear. If $AB = 3x + 7$, $BC = 6x + 3$, and $AC = 12x - 11$, Find AC.

$$3x + 7 + 6x + 3 = 12x - 11$$

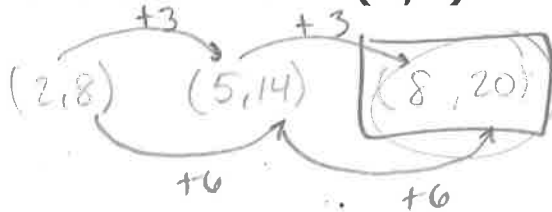
$$9x + 10 = 12x - 11$$

$$21 = 3x \quad x = 7$$

$$AC = 12(7) - 11$$

$$AC = 73$$

2. B is the midpoint of AC. Find the coordinate of C if A is at (2,8) and B is at (5,14)



Sep 14-4:04 PM

Use the Venn diagram to determine whether the statement is true or false. Justify your answer. Assume that no region of the Venn diagram is empty.

a. If an item has Property B, then it has Property A. *True*

b. If an item has Property A, then it has Property B. *False*

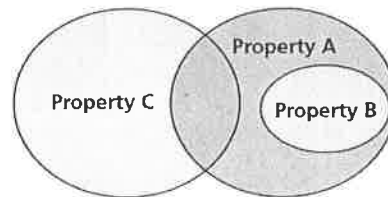
c. If an item has Property A, then it has Property C. *False*

d. Some items that have Property A do not have Property B. *True*

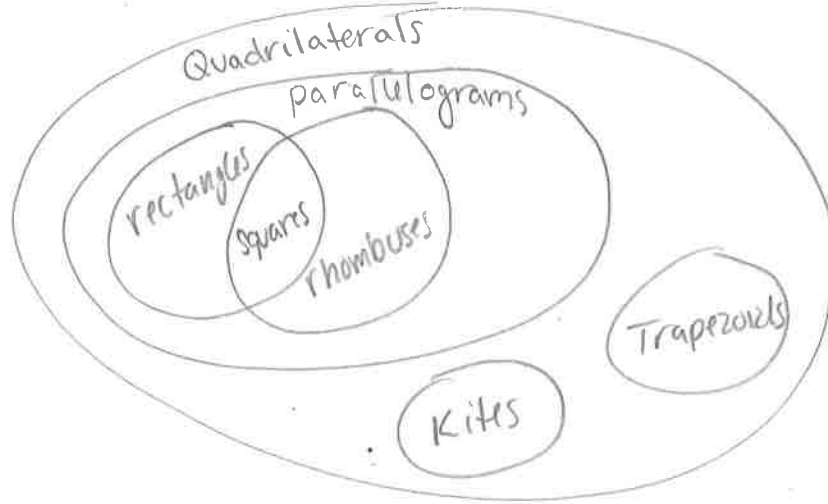
e. If an item has Property C, then it does not have Property B. *True*

f. Some items have both Properties A and C. *True*

g. Some items have both Properties B and C. *False*



Work with a partner. Draw a Venn diagram that shows the relationship between different types of quadrilaterals: squares, rectangles, parallelograms, trapezoids, rhombuses, and kites. Then write several conditional statements that are shown in your diagram, such as "If a quadrilateral is a square, then it is a rectangle."



Exploration 3

Core Concept

Inductive Reasoning

A **conjecture** is an unproven statement that is based on observations. You use **inductive reasoning** when you find a pattern in specific cases and then write a conjecture for the general case.

Write a conjecture about the pattern. Then use your conjecture to draw the 10th object in the pattern.

1 2 3 4 5 6 7 8 9 10

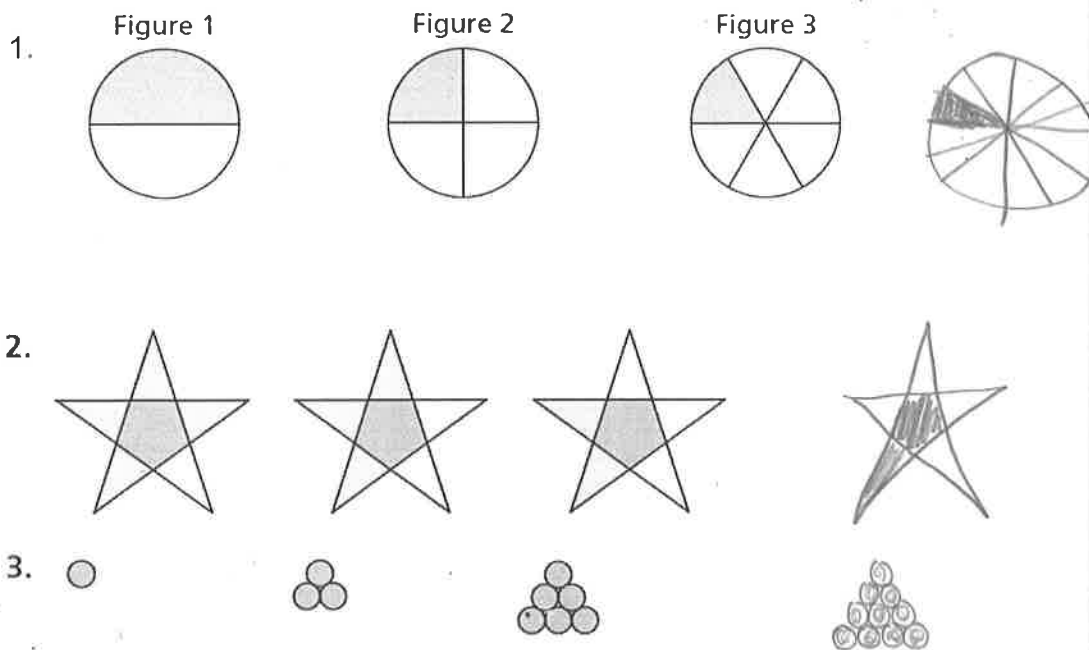
a.

b.

c.

Core Concept

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.



Example 1

1. Numbers such as 3, 4, and 5 are called *consecutive integers*. Make and test a conjecture about the sum of any three consecutive integers.

$$1 + 2 + 3 = 6$$

$$7 + 8 + 9 = 24$$

$$-4 + -5 + -6 = -15$$

The sum is 3 times the middle #

2. Make and test a conjecture about the sign of the product of any three negative integers.

$$(-2)(-2)(-2) = -8$$

$$(-3)(-3)(-3) = -27$$

Negative

3. Make and test a conjecture about the sum of any five consecutive integers.

$$1 + 2 + 3 + 4 + 5 = 15$$

$$-2 + -3 + -4 + -5 + -6 = -20$$

$$6 + 7 + 8 + 9 + 10 = 40$$

The sum is five times the middle #

Example 2

Core Concept

Counterexample

To show that a conjecture is true, you must show that it is true for all cases. You can show that a conjecture is false, however, by finding just one *counterexample*. A **counterexample** is a specific case for which the conjecture is false.

Core Concept

A student makes the following conjectures. Find a counterexample to disprove the student's conjectures.

1. Conjecture: The sum of two numbers is always more than the greater number.

$$\begin{aligned} -2 + -5 &= -7 \\ -7 &\not> -5 \end{aligned}$$

2. Conjecture: The value of x^2 is always greater than the value of x .

$$\begin{aligned} \text{If } x=1, \text{ then } (1)^2 &= 1 \\ 1 &\not> 1 \end{aligned}$$

3. Conjecture: The sum of two numbers is always greater than their difference.

$$\begin{aligned} 2 + (-3) &= -1 & -1 &\not> 5 \\ 2 - (-3) &= 5 \end{aligned}$$

Example 3

Deductive Reasoning

Deductive reasoning uses facts, definitions, accepted properties, and the laws of logic to form a logical argument. This is different from *inductive reasoning*, which uses specific examples and patterns to form a conjecture.

Decide whether inductive reasoning or deductive reasoning is used to reach the conclusion. Explain your reasoning.

a. Each time Monica kicks a ball up in the air, it returns to the ground. So, the next time Monica kicks a ball up in the air, it will return to the ground. Inductive

b. All reptiles are cold-blooded. Parrots are not cold-blooded. Sue's pet parrot is not a reptile. Deductive

Core Concept

Laws of Logic

Law of Detachment

If the hypothesis of a true conditional statement is true, then the conclusion is also true.

Applies a conditional statement to a specific example.

To be valid: The conditional and the example both have to be in the form $p \rightarrow q$.

Core Concept

If two segments have the same length, then they are congruent. You know that $BC = XY$. Using the Law of Detachment, what statement can you make?

$$\overline{BC} \cong \overline{XY}$$

If $90^\circ < m\angle R < 180^\circ$, then $\angle R$ is obtuse. The measure of $\angle R$ is 155° . Using the Law of Detachment, what statement can you make?

$\angle R$ is obtuse

Example 4

Is the conjecture valid or invalid based on Law of Detachment?

Given: If a team wins four games in the World Series, then the team wins the series. The Red Sox won four games in the 2004 World Series.

Conjecture: The Red Sox won the 2004 World Series.

valid

Law of Syllogism

If $p \rightarrow q$ and $q \rightarrow r$ are true statements, then $p \rightarrow r$ is a true statement.

If hypothesis p , then conclusion q .

If hypothesis q , then conclusion r .

If hypothesis p , then conclusion r .

↗ If these statements are true,

← then this statement is true.

Sep 14-4:18 PM

If possible, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements.

a. If $x^2 > 25$, then $x^2 > 20$.

If $x > 5$, then $x^2 > 25$.

If $x > 5$, then $x^2 > 20$

b. If a polygon is regular, then all angles in the interior of the polygon are congruent.

If a polygon is regular, then all its sides are congruent.

Not possible

Example 5

Use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements.

If you get an A on your math test, then you can go to the movies.

If you go to the movies, then you can watch your favorite actor.

If you get an A, then you can watch your favorite actor.

Sep 14-4:23 PM

Determine if the conjecture is valid by the Law of Syllogism.

Given: If an animal is a mammal, then it has hair. If an animal is a dog, then it is a mammal.

Conjecture: If an animal is a dog, then it has hair.

valid

Sep 14-4:31 PM

Determine if the conjecture is valid by the Law of Syllogism.

Given: If Lulu Rae comes out with a new clothing item, Mrs. Lyon will buy it. If Mrs. Lyon buys Lulu Rae, then the other teachers will roll their eyes.

Conjecture: If the other teachers roll their eyes, then Lulu Rae came out with a new clothing item.

Invalid

Sep 14-4:31 PM

Using Inductive *and* Deductive reasoning.

1. What conclusion can you make about the product of an even integer and any other integer?

a. Use inductive reasoning to make a conjecture.

$$2(5) = 10 \quad 2(7) = 14 \quad 2(4) = 8 \quad 2(-3) = -6$$

product is even

b. Use deductive reasoning to show the conjecture is true

$$(2n)(m) = 2nm$$

2. Use inductive reasoning to make a conjecture about the sum of a number and itself. Then use deductive reasoning to show that the conjecture is true.

$$2+2=4$$

$$3+3=6$$

$$5+5=10$$

$$n+n=2n$$

Example 6

Decide whether inductive reasoning or deductive reasoning is used to reach the conclusion. Explain your reasoning.

All multiples of 8 are divisible by 4.

64 is a multiple of 8.

So, 64 is divisible by 4.

Deductive

Monitoring Progress 10-11

Homework

pg. 80 # 4, 6, 8, 9-19, 23-26, 29-32, (44?)

Sep 14-4:35 PM