

## 2.5 - Proving Statements about Segments and Angles

Find the complement and the supplement of the angle measure.

1. $59^\circ$ $90 - 59 = 31^\circ$ $180 - 59 = 121^\circ$	2. $20^\circ$ $90 - 20 = 70^\circ$ $180 - 20 = 160^\circ$	3. $53^\circ$ $90 - 53 = 37^\circ$ $180 - 53 = 127^\circ$
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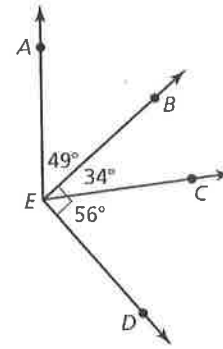
Use the figure.

1. Name a pair of adjacent complementary angles.

2. Find  $m\angle BED$ .

$$m\angle BED = 90^\circ$$

$\angle BEC$  and  $\angle CED$



Warm Up

### Essential Question

How can you prove a mathematical statement?

A **proof** is an argument that uses logic, definitions, properties, and previously proven statements (deductive reasoning) to show that a conclusion is true.

**For the reasons, you will use definitions including, but not limited to:**

Def. of Complementary Angles

Def. of Supplementary Angles

Def. of Congruence

Def. of Right Angle

Def. of Straight Angle

All of the properties of congruence or equality

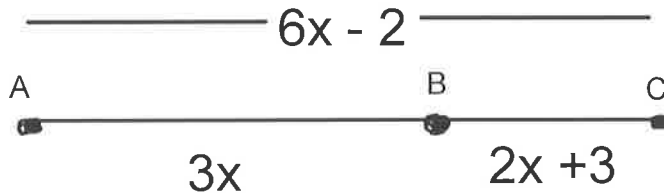
Any new theorems we learn

Essential Question

Solve for x. Write a justification for each step.

Statement	Reason
$m\angle ABD = (3x + 5)$ ; $m\angle DBC = (6x - 16)$ $m\angle ABC = 8x$	Given
$m\angle ABD + m\angle DBC = m\angle ABC$	Angle Addition Postulate
$3x + 5 + 6x - 16 = 8x$	Substitution
$9x - 11 = 8x$	Simplify
$\begin{array}{r} 9x - 11 \\ -9x \quad -9x \\ \hline -11 = -x \end{array}$	Subtraction POE
$\begin{array}{r} -11 = -x \\ \hline -1 \quad -1 \end{array}$	Simplify
$11 = x$	Division POE
$x = 11$	Simplify

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Solve for x. Justify each step.

Statements	Reasons
$AB = 3x$ , $BC = 2x + 3$ , $AC = 6x - 2$	Given
$AB + BC = AC$	Segment Addition Postulate
$3x + 2x + 3 = 6x - 2$	Substitution
$5x + 3 = 6x - 2$	Simplify
$\begin{array}{r} 5x + 3 = 6x - 2 \\ -5x \quad -5x \\ \hline 3 = x - 2 \end{array}$	Subtraction POE
$\begin{array}{r} 3 = x - 2 \\ +2 \quad +2 \\ \hline 5 = x \end{array}$	Simplify
$x = 5$	Addition POE
$x = 5$	Simplify

Symmetri  
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Given: B is the midpoint of AC.  
Find the length of AB. Justify each step.

Statements  
 $AB = 2x, BC = 4x - 8$   
 B is mdpt of AC  
 $\overline{AB} \cong \overline{BC}$   
 $AB = BC$   
 $2x = 4x - 8$   
 $\begin{array}{r} -4x \quad -4x \\ \hline -2x = -8 \end{array}$   
 $\begin{array}{r} -2x = -8 \\ \hline -2 \quad -2 \\ \hline x = 4 \end{array}$

Reasons  
 Given  
 Given  
 Definition of midpoint  
 Def. of  $\cong$   
 Substitution  
 Subtraction POE  
 Simplify  
 Division POE  
 Simplify

$AB = 2(4)$

Sep 25 3:23 PM Substitution

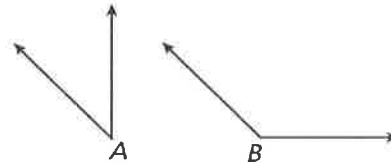
$AB = 8$

Simplify

Write a justification for each step, given that  $\angle A$  and  $\angle B$  are supplementary and  $m\angle A = 45^\circ$ . Find  $m\angle B$ .

Statement  
 $\angle A + \angle B$  are supp.  $\angle$ 's  
 $m\angle A = 45^\circ$   
 $m\angle A + m\angle B = 180$   
 $\begin{array}{r} 45 + m\angle B = 180 \\ -45 \quad \quad -45 \\ \hline m\angle B = 135^\circ \end{array}$

Reasons  
 Given  
 Given  
 Def. of supplementary  
 sub  
 subtraction POE  
 Simplify

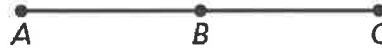


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**Work with a partner.** Four steps of a proof are shown. Write the reasons for each statement.

**Given**  $AC = AB + AB$

**Prove**  $AB = BC$

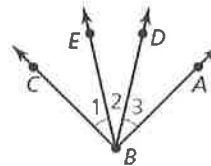


STATEMENTS	REASONS
1. $AC = AB + AB$	1. Given
2. $AB + BC = AC$	2. Segment Addition Postulate
3. $AB + AB = AB + BC$	3. Substitution
4. $AB = BC$	4. Subtraction PDE

Exploration 1

**Given**  $m\angle 1 = m\angle 3$

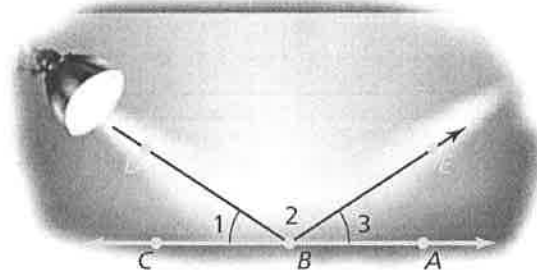
**Prove**  $m\angle EBA = m\angle CBD$



STATEMENTS	REASONS
1. $m\angle 1 = m\angle 3$	1. Given
2. $m\angle EBA = m\angle 2 + m\angle 3$	2. Angle Addition Postulate (Post.1.4)
3. $m\angle EBA = m\angle 2 + m\angle 1$	3. Substitution Property of Equality
4. $m\angle EBA = m\angle 1 + m\angle 1 + m\angle 2$	4. Commutative Property of Addition
5. $m\angle 1 + m\angle 2 = m\angle CBD$	5. Angle Addition Postulate (Post.1.4)
6. $m\angle EBA = m\angle CBD$	6. Transitive Property of Equality

Exploration 2

Write a two-column proof for the situation in Example 4 from the Section 2.4 lesson.



**Given**  $m\angle 1 = m\angle 3$

**Prove**  $m\angle DBA = m\angle EBC$

<u>Statement</u>	<u>Reason</u>
$m\angle 1 = m\angle 3$	Given
$m\angle 1 + m\angle 2 = m\angle CBE$	Angle Add. Postulate
$m\angle 2 + m\angle 3 = m\angle DBA$	Angle Add. Postulate
$m\angle 2 + m\angle 1 = m\angle DBA$	Sub
$m\angle 1 + m\angle 2 = m\angle DBA$	Commutative POE
$m\angle CBE = m\angle DBA$	Transitive POE

Example 1

**Given**  $T$  is the midpoint of  $\overline{SU}$ .

**Prove**  $x = 5$



STATEMENTS	REASONS
1. $T$ is the midpoint of $\overline{SU}$ .	1. <u>Given</u>
2. $\overline{ST} \cong \overline{TU}$	2. Definition of midpoint
3. $ST = TU$	3. Definition of congruent segments
4. $7x = 3x + 20$	4. <u>Substitution</u>
5. <u><math>4x = 20</math></u>	5. Subtraction Property of Equality
6. $x = 5$	6. <u>DIVISION POE</u>

A **theorem** is any statement that you can prove.

Once you have proven a theorem, you can use it as a reason in later proofs.

## Theorems

### Theorem 2.1 Properties of Segment Congruence

Segment congruence is reflexive, symmetric, and transitive.

**Reflexive** For any segment  $AB$ ,  $\overline{AB} \cong \overline{AB}$ .

**Symmetric** If  $\overline{AB} \cong \overline{CD}$ , then  $\overline{CD} \cong \overline{AB}$ .

**Transitive** If  $\overline{AB} \cong \overline{CD}$  and  $\overline{CD} \cong \overline{EF}$ , then  $\overline{AB} \cong \overline{EF}$ .

*Proofs* Ex. 11, p. 103; Example 3, p. 101; Chapter Review 2.5 Example, p. 118

### Theorem 2.2 Properties of Angle Congruence

Angle congruence is reflexive, symmetric, and transitive.

**Reflexive** For any angle  $A$ ,  $\angle A \cong \angle A$ .

**Symmetric** If  $\angle A \cong \angle B$ , then  $\angle B \cong \angle A$ .

**Transitive** If  $\angle A \cong \angle B$  and  $\angle B \cong \angle C$ , then  $\angle A \cong \angle C$ .

*Proofs* Ex. 25, p. 118; 2.5 Concept Summary, p. 102; Ex. 12, p. 103

Theorem

Name the property that the statement illustrates.

a. If  $\angle T \cong \angle V$  and  $\angle V \cong \angle R$ , then  $\angle T \cong \angle R$ .

Transitive POC

b. If  $\overline{JL} \cong \overline{YZ}$ , then  $\overline{YZ} \cong \overline{JL}$ .

Symmetric POC

c.  $\overline{GH} \cong \overline{GH}$

Reflexive POC

d. If  $\angle K \cong \angle P$ , then  $\angle P \cong \angle K$ .

Symmetric POC

Example 2

Prove this property of midpoints: If you know that  $M$  is the midpoint of  $\overline{AB}$ , prove that  $AB$  is two times  $AM$  and  $AM$  is one-half  $AB$ .

**Given:**  $M$  is the midpoint of  $\overline{AB}$ .

**Prove:**  $AB = 2AM$ ,  $AM = \frac{1}{2}AB$



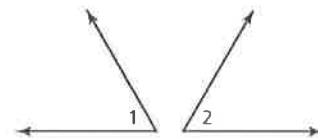
<u>Statement</u>	<u>Reason</u>
$M$ is midpoint of $\overline{AB}$	Given
$\overline{AM} \cong \overline{MB}$	Def. of midpoint
$AM = MB$	Def. of $\cong$
$AM + MB = AB$	Segment Add. Postulate
$AM + AM = AB$	Sub
$2AM = AB$	Simplify
$\frac{2AM}{2} = \frac{AB}{2}$	Division PDE
$AM = \frac{1}{2}AB$	Simplify

Example 4

### Concept Summary

#### Writing a Two-Column Proof

In a proof, you make one statement at a time until you reach the conclusion. Because you make statements based on facts, you are using deductive reasoning. Usually the first statement-and-reason pair you write is given information.



Copy or draw diagrams and label given information to help develop proofs. Do not mark or label the information in the Prove statement on the diagram.

#### Proof of the Symmetric Property of Angle Congruence

Given  $\angle 1 \cong \angle 2$

Prove  $\angle 2 \cong \angle 1$

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	1. Given
2. $m\angle 1 = m\angle 2$	2. Definition of congruent angles
3. $m\angle 2 = m\angle 1$	3. Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	4. Definition of congruent angles

statements based on facts that you know or on conclusions from deductive reasoning

The number of statements will vary.

Remember to give a reason for the last statement.

definitions, postulates, or proven theorems that allow you to state the corresponding statement

Concept Summary

Homework:

pg. 103 #3-10, 13, 14

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