4.1 - Translations

Bellwork

Translate point *P*. State the coordinates of *P*′.

1. $P(-4, 4)$; 2 units down, 2 units right

P' (-2, 2)

2. $P(-3, -2)$; 3 units right, 3 units up

P' (0, 1)

3. $P(2, 2)$; 2 units down, 2 units right

P' (4, 0)

Warm Up 1-3

A **transformation** is a function that moves or changes a figure in some way to produce a new figure called an **image**. Another name for the original figure is the **preimage**.

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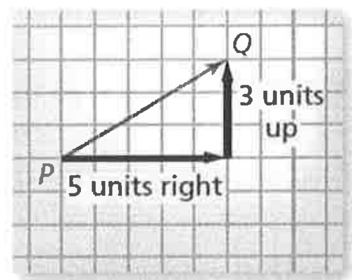
A **Vector** is a quantity that has both magnitude (length) and direction.

Initial Point - the starting point

Terminal Point - the ending point

The vector is named $\overrightarrow{PQ}$, which is read as "vector PQ." The **horizontal component** of $\overrightarrow{PQ}$ is 5, and the **vertical component** is 3.

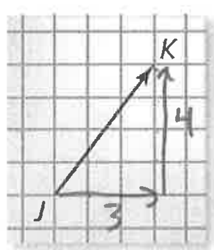


Component Form of a Vector <x, y>
Lists the horizontal and vertical change from the initial point to the terminal point

Core Concept

In the diagram, name the vector and write its component form.

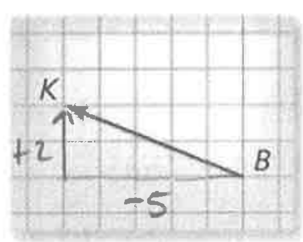
a)



$\overrightarrow{JK}$

$\langle 3,4 \rangle$

b)



$\overrightarrow{BK}$

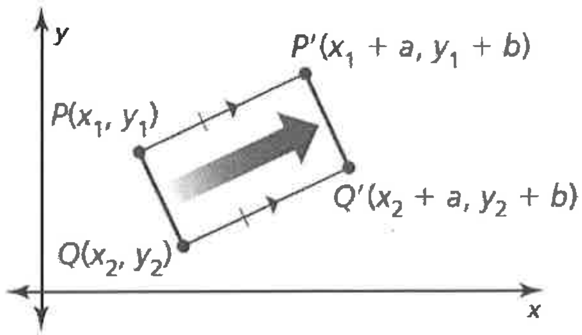
$\langle -5,2 \rangle$

Example 1

A **translation** (Slide)
- a transformation where all the points of a figure are moved the same distance in the same direction.

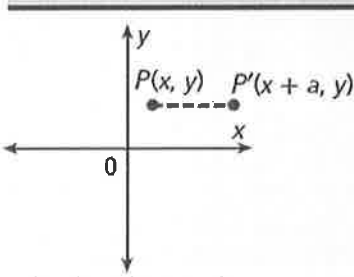
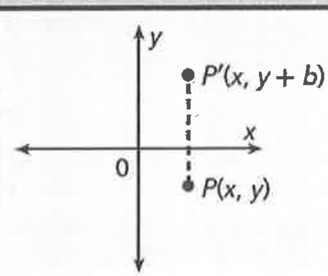
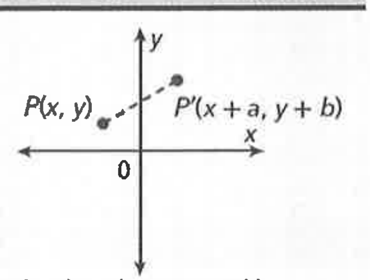
- *maps* (or moves) the points P and Q of a plane figure along vector <a, b> to the points P' and Q' so that one of the following is true:

- $PP' = QQ'$ and $\overline{PP'} \parallel \overline{QQ'}$, or
- $PP' = QQ'$ and $\overline{PP'}$ and $\overline{QQ'}$ are collinear.



Core Concept

Translations in the Coordinate Plane

HORIZONTAL TRANSLATION ALONG VECTOR $\langle a, 0\rangle$	VERTICAL TRANSLATION ALONG VECTOR $\langle 0, b\rangle$	GENERAL TRANSLATION ALONG VECTOR $\langle a, b\rangle$
		
$(x, y) \rightarrow (x + a, y)$	$(x, y) \rightarrow (x, y + b)$	$(x, y) \rightarrow (x + a, y + b)$

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The vertices of $\triangle ABC$ are $A(0, 3)$, $B(2, 4)$, and $C(1, 0)$. Translate $\triangle ABC$ using the vector $\langle 5, -1\rangle$.

$(x, y) \rightarrow (x+5, y-1)$

$A\ (0,3) \rightarrow A'\ (5,2)$

$B\ (2,4) \rightarrow B'\ (7,3)$

$C\ (1,0) \rightarrow C'\ (6,-1)$

The vertices of $\triangle LMN$ are $L(2, 2)$, $M(5, 3)$, and $N(9, 1)$. Translate $\triangle LMN$ using the vector $\langle -2, 6\rangle$.

$(x,y) \rightarrow (x-2, y+6)$

$L\ (2,2) \rightarrow L'\ (0,8)$

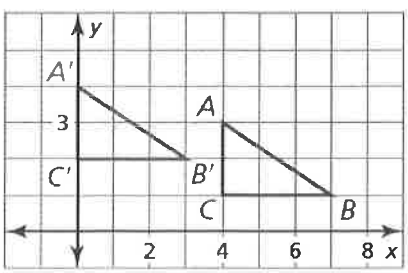
$M\ (5,3) \rightarrow M'\ (3,9)$

$N\ (9,1) \rightarrow N'\ (7,7)$

You can also express a translation along a vector <a,b> using a rule, which has the notation (x,y) ⟶(x+a, y+b).

Example 2

Write a rule for the translation of $\triangle ABC$ to $\triangle A'B'C'$.



coordinate rule

$(x,y) \longrightarrow (x-4, y+1)$

component form of a vector

$\langle -4, 1\rangle$

Example 3

Quadrilateral $ABCD$ has vertices $A(-1, 2)$, $B(-1, 5)$, $C(4, 6)$, and $D(4, 2)$. Find the coordinates of the image after the translation $(x, y) \rightarrow (x + 3, y - 1)$.

$A(-1,2) \rightarrow A'(2,1)$

$B(-1,5) \rightarrow B'(2,4)$

$C(4,6) \rightarrow C'(7,5)$

$D(4,2) \rightarrow D'(7,1)$

$\triangle RST$ has vertices $R(2, 2)$, $S(5, 2)$, and $T(3, 5)$. Find its image after the translation $(x, y) \rightarrow (x + 1, y + 2)$.

$R(2,2) \rightarrow R'(3,4)$

$S(5,2) \rightarrow S'(6,4)$

$T(3,5) \rightarrow T'(4,7)$

Example 4

A **rigid motion (also called an Isometry)** is a transformation that preserves length and angle measure.

Postulate

Postulate 4.1 Translation Postulate

A translation is a rigid motion.

Theorem

Theorem 4.1 Composition Theorem

The composition of two (or more) rigid motions is a rigid motion.

Proof Ex. 35, p. 180

Postulate

$\overline{TU}$ has endpoints $T(1, 2)$ and $U(4, 6)$. Find its image after the composition.

Translation: $(x, y) \rightarrow (x - 2, y - 3)$

Translation: $(x, y) \rightarrow (x - 4, y + 5)$

$T'(-1,-1) \rightarrow T''(-5,4)$

$U'(2,3) \rightarrow U''(-2,8)$

$\overline{VW}$ has endpoints $V(-6, -4)$ and $W(-3, 1)$. Find its image after the composition.

Translation: $(x, y) \rightarrow (x + 3, y + 1)$

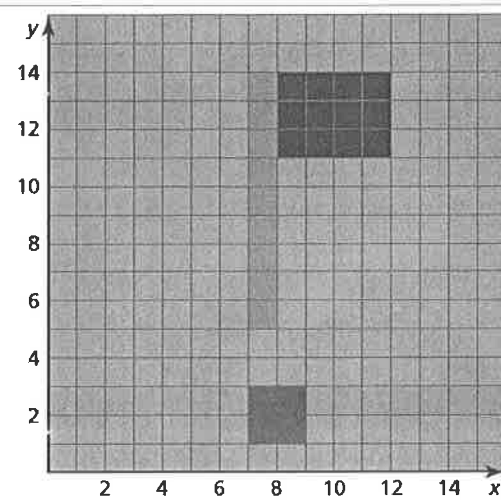
Translation: $(x, y) \rightarrow (x - 6, y - 4)$

$V'(-3,-3) \rightarrow V''(-9,-7)$

$W'(0,2) \rightarrow W''(-6,-2)$

Monitoring Progress 5

You are designing a favicon for a golf website. In an image-editing program, you move the red rectangle 2 units left and 3 units down. Then you move the red rectangle 1 unit right and 1 unit up. Rewrite the composition as a single translation.



$(x,y) \rightarrow (x-1, y-2)$

Example 2

you move the gray square 2 units right and 3 units up. Then you move the gray square 1 unit left and 1 unit down. Rewrite the composition as a single transformation. $(x,y) \rightarrow (x+1, y+2)$

Example 6

Homework:

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