4.2 - Reflections

Reflect point *P*. State the coordinates of *P'*.

1. *P*(−5, 3); reflection in *y*-axis P'(5,3)
2. *P*(−4, −3); reflection in *y*-axis P'(4,-3)
3. *P*(−1, 1); reflection in *x*-axis P'(-1,-1)
4. *P*(4, 6); reflection in *x*-axis P'(4,-6)

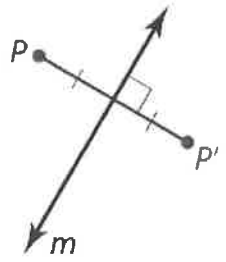
Warm Up 1-3

Reflections

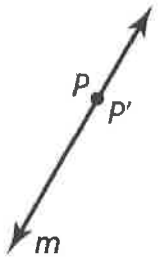
A **reflection** is a transformation that uses a line like a mirror to reflect a figure. The mirror line is called the **line of reflection**.

A reflection in a line m maps every point P in the plane to a point P', so that for each point one of the following properties is true.

- If P is not on m, then m is the perpendicular bisector of $\overline{PP'}$, or
- If P is on m, then $P = P'$.



point *P* not on *m*



point *P* on *m*

Postulate

Postulate 4.2 Reflection Postulate

A reflection is a rigid motion.

Core Concept

Find the image of $\triangle ABC$ with vertices $A(1, 3)$, $B(5, 2)$, and $C(2, 1)$ after the reflection described.

1. In the line $x = 3$

A'(5, 3) B'(1, 2) C'(4, 1)

2. In the line $y = 1$

A'(1, -1) B'(5, 0) C'(2, 1)

3. In the line $x = 4$

A'(7, 3) B'(3, 2) C'(6, 1)

4. In the line $x = -3$

A'(-7, 3) B'(-11, 2) C'(-8, 1)

5. In the line $y = 2$

A'(1, 1) B'(5, 2) C'(2, 3)

6. In the line $y = -1$

A'(1, -5) B'(5, -4) C'(2, -3)

Example 1

Coordinate Rules for Reflections

In the x-axis: (a, b) → (a, -b)

In the y-axis: (a, b) → (-a, b)

In the line y = x: (a, b) → (b, a)

In the line y = -x: (a, b) → (-b, -a)

Core Concept

$\overline{FG}$ has endpoints $F(-1, 2)$ and $G(1, 2)$. Find its image after a reflection:

1. in the line $y = x$. F'(2,-1) G'(2,1)

2. in the line $y = -x$ F'(-2,1) G'(-2,-1)

Example 2

The vertices of $\triangle JKL$ are $J(1, 3)$, $K(-2, 4)$, and $L(3, 1)$. Find the image after a reflection:

1. in the x-axis. J'(1,-3) K'(-2,-4) L'(3,-1)

2. in the y-axis. J'(-1,3) K'(2,4) L'(-3,1)

3. in the line $y = x$. J'(3,1) K'(4,-2) L'(1,3)

4. in the line $y = -x$. J'(-3,-1) K'(-4,2) L'(-1,-3)

Monitoring Progress 5-6

A **glide reflection** is a transformation involving a translation followed by a reflection in which every point P is mapped to a point P" by the following steps:

1. Translation maps P to P'

2. A reflection in line k parallel to the direction of the translation maps P' to P".

Nov 7-6:45 AM

Find the image of $\triangle ABC$ with vertices $A(3, 2)$, $B(6, 3)$, and $C(7, 1)$ after the glide reflection.

Translation: $(x, y) \rightarrow (x - 12, y)$

Reflection: in the x-axis

$A'(-9, 2) \longrightarrow A''(-9, -2)$

$B'(-6, 3) \longrightarrow B''(-6, -3)$

$C'(-5, 1) \longrightarrow C''(-5, -1)$

Translation: $(x, y) \rightarrow (x , y - 4)$

Reflection: in the y-axis

$A'(3, -2) \longrightarrow A''(-3, -2)$

$B'(6, -1) \longrightarrow B''(-6, -1)$

$C'(7, -3) \longrightarrow C''(-7, -3)$

Example 4

Line symmetry – if a figure can be mapped onto itself by a reflection in a line. This line of reflection is a **line of symmetry.**

Determine the number of lines of symmetry for the figure.

a. 2

b. 6

c. 1

d 2

e 5

f 1

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Homework:

pg. 186 #2-20 Evens, 21-25

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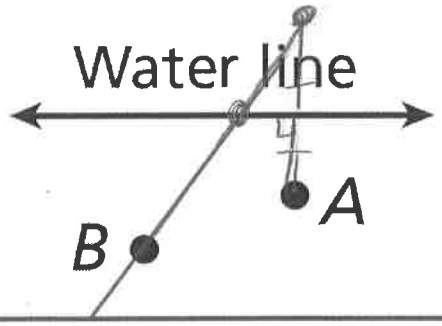
Finding the shortest path...

Two buildings located at *A* and *B* are to be connected to the same point on the water line. Where should they connect so that the least amount of pipe will be used?



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Find Point C on the x-axis so AC + BC is a minimum

Ex. 1 A (2, 5), B(7, 3)

Reflect A over x-axis

$A'(2,-5)$

Find equation of line through (2,-5) and (7,3)

$+5\left(\frac{2,-5}{7,3}\right)+8$

$m=\frac{8}{5}$ (7,3)

$y=mx+b$

$3=\frac{8}{5}(7)+b$

$\frac{15}{3}=\frac{56}{5}+b$

$b=-\frac{41}{5}$

$y=\frac{8}{5}x-\frac{41}{5}$

On x-axis, y=0

So, "

$0=\frac{8}{5}x-\frac{41}{5}$

$\frac{41}{5}=\frac{8x}{5}$

$41=8x$

$x=5.125$

$(5.125,0)$

Ex. 2 A (2, 7), B(8, -2)

* Already on opposite sides of x-axis → No need to reflect any points

$+6\left(\frac{2,7}{8,-2}\right)-9$

$m=\frac{-9}{6}=-\frac{3}{2}$

$y=mx+b$

$7=-\frac{3}{2}(2)+b$

$7=-3+b$

$10=b$

$y=-\frac{3}{2}x+10$

$0=-\frac{3}{2}x+10$

$-10=-\frac{3}{2}x$

$\frac{20}{3}=x$

$\left(\frac{20}{3},0\right)$ or $(6.67,0)$

Practice Problems: pg. 187 #29-32

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