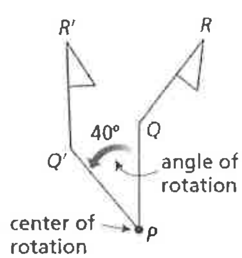
4.3 Rotations

Warm Up 1-3

Rotation - a transformation that turns a figure around a fixed point, called the center of rotation.



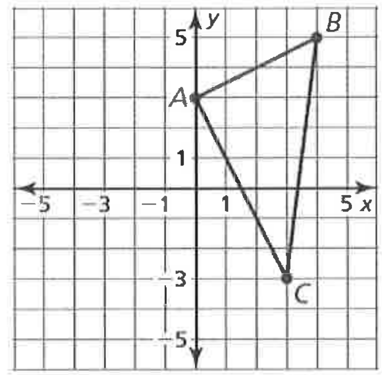
Postulate 4.3 Rotation Postulate

A rotation is a rigid motion.

Unless otherwise stated, all rotations in this book are counterclockwise.

Core Concept

Work with a partner.



a. The point (x, y) is rotated 90° counterclockwise about the origin. Write a rule to determine the coordinates of the image of (x, y).

b. Use the rule you wrote in part (a) to rotate $\triangle ABC$ 90° counterclockwise about the origin. What are the coordinates of the vertices of the image, $\triangle A'B'C'$?

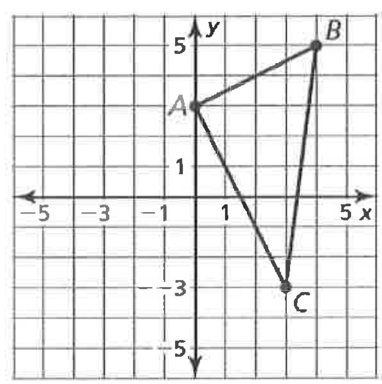
A' (-3,0)

B' (-5,4)

C' (3,3)

Exploration 2

Work with a partner.



a. The point (x, y) is rotated 180° counterclockwise about the origin. Write a rule to determine the coordinates of the image of (x, y). Explain how you found the rule.

b. Use the rule you wrote in part (a) to rotate $\triangle ABC$ (from Exploration 2) 180° counterclockwise about the origin. What are the coordinates of the vertices of the image, $\triangle A'B'C'$?

A' (0,-3)

B' (-4,-5)

C' (-3,3)

Exploration 3

Coordinate Rules for Rotations about the Origin

When a point (a, b) is rotated counterclockwise about the origin, the following are true.

- For a rotation of 90°, $(a, b) \rightarrow (-b, a)$.
- For a rotation of 180°, $(a, b) \rightarrow (-a, -b)$.
- For a rotation of 270°, $(a, b) \rightarrow (b, -a)$.

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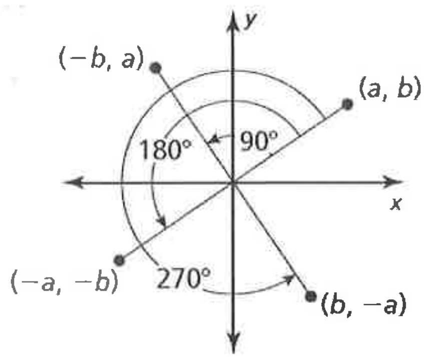

Core Concept

Rotate point *P* counterclockwise about the origin by the given angle. State the coordinates of *P*′.

1. $P(4, 2)$; 90° → $(-b, a)$ $P'(-2, 4)$

2. $P(3, 0)$; 90° → $(-b, a)$ $P'(0, 3)$

3. $P(6, 0)$; 180° → $(-a, -b)$ $P'(-6, 0)$

4. $P(2, 6)$; 180° → $(-a, -b)$ $P'(-2, -6)$

5. $P(-2, 0)$; 270° → $(b, -a)$ $P'(0, 2)$

6. $P(4, 0)$; 270° → $(b, -a)$ $P'(0, -4)$

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Compositions of Transformations

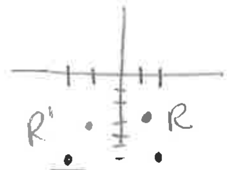
1. Given $\overline{RS}$ with endpoints $R(1, -3)$ and $S(2, -6)$, find its image after the composition.

Reflection: in the y-axis

Rotation: 90° about the origin

R'(-1,-3) → (-b,a) → R''(3,-1)

S'(-2,-6) → (-b,a) → S''(6,-2)



2. Given $\overline{RS}$ from Example 3. Perform the rotation first, followed by the reflection. Does the order of the transformations matter? Explain.

Rot. 90°

R(1,-3) → (-b,a) → R'(3,1) Reflect R'' (-3,1)

S(2,-6) → (-b,a) S'(6,2) S'' (-6,2)

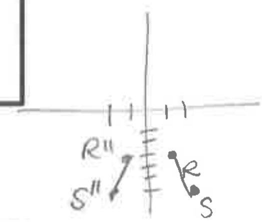
Normally, it will be true.

3. WHAT IF? In Example 3, $\overline{RS}$ is reflected in the x-axis and rotated 180° about the origin. Graph $\overline{RS}$ and its image after the composition.

R(1,-3) x-axis reflect. R'(1,3) 180° ROT. (-a,-b) R''(-1,-3)

S(2,-6) S'(2,6) (-a,-b) S''(-2,-6)

Example 3



4. Given $\overline{AB}$ with endpoints $A(-4, 4)$ and $B(-1, 7)$, find its image after the composition.
-2 -1

Translation: $(x, y) \rightarrow (x - 2, y - 1)$

Rotation: 90° about the origin

A'(-6,3) → (-b,a) A''(-3,-6)

B'(-3,6) → (-b,a) B''(-6,-3)

5. Given $\triangle TUV$ with vertices $T(1, 2)$, $U(3, 5)$, and $V(6, 3)$, find its image after the composition.

Rotation: 180° about the origin

Reflection: in the x-axis

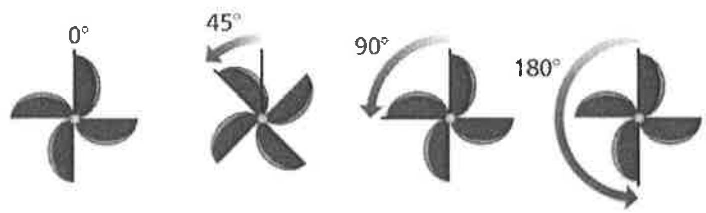
T(1,2) → (-a,-b) - T'(-1,-2) x-axis → T''(-1,2)

U(3,5) → - U'(-3,-5) U''(-3,5)

V(6,3) → - V'(-6,-3) V''(-6,3)

Monitoring Progress 5

A figure in the plane has **rotational symmetry** when the figure can be mapped onto itself by a rotation of 180 degrees or less about the center of the figure. This point is called the **center of symmetry.**



Order - The number of times you rotate it to get it back to the original

360/Order = Angle of Rotational Symmetry

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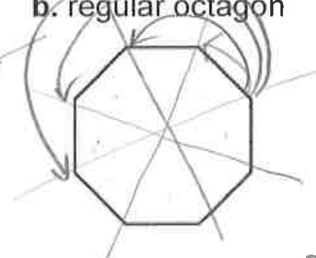
Does the figure have rotational symmetry? If so, describe any rotations that map the figure onto itself.

a. parallelogram

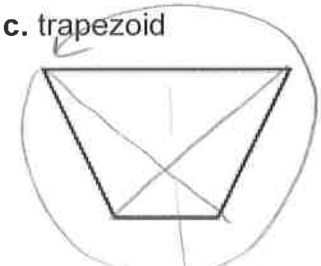
Yes, 180° rotation

b. regular octagon



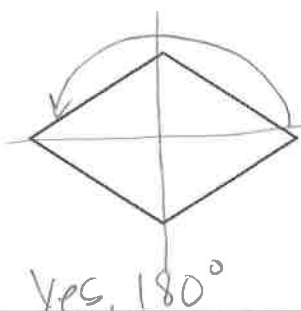
Yes, 45°, 90°, 135°, 180°

c. trapezoid



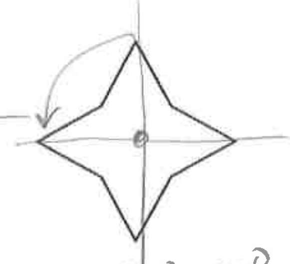
No, it doesn't.

d. rhombus



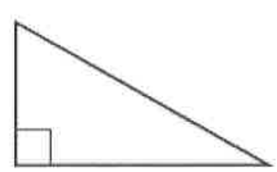
Yes, 180°

e. octagon



Yes, 90°, 180°

f. right triangle



No

Example 4

Homework

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