

4.4 - Congruence and Transformations

Find the coordinates of point P along the directed line segment AB so that AP to PB is the given ratio.

1. $A(4, -3), B(9, -1); 2$ to $3 = \frac{2}{5}$

2. $A(-1, -5), B(7, 0); 4$ to 1

① $+5 \begin{pmatrix} 4, -3 \\ 9, -1 \end{pmatrix} + 2$

$\frac{2 \cdot 2/5}{5 \cdot 2/5} = \frac{0.8 \Delta y}{2 \Delta x}$

$P(4+2, -3+0.8)$

$P(6, -2.2)$

② $+8 \begin{pmatrix} -1, -5 \\ 7, 0 \end{pmatrix} + 5$

$\frac{5 \cdot 4/5}{8 \cdot 4/5} = \frac{4 \Delta y}{6.4 \Delta x}$

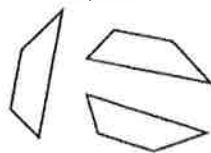
$(-1+6.4, -5+4)$

$P(5.4, -1)$

Cumulative Warm Up

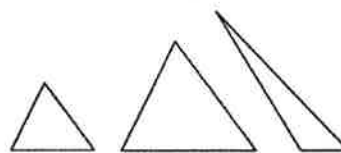
Congruent Figures have the same size and shape. All sides and angles are congruent.

Congruent



same size and shape

Not congruent



different sizes or shapes

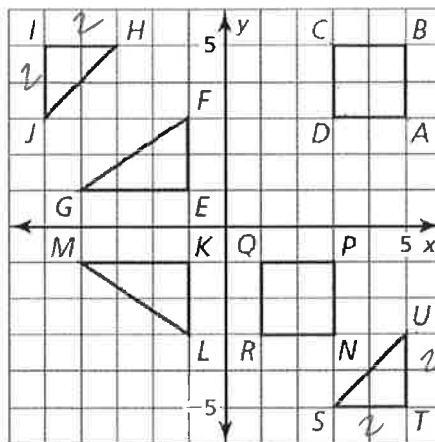
Two geometric figures are congruent iff there is a rigid motion or composition of rigid motions that maps one of the figures onto the other.

Identify any congruent figures in the coordinate plane. Explain.

$\triangle JIH \cong \triangle UTS$ are 180° rotations of each other

$\triangle EFG \cong \triangle KLM$
- reflection over the x-axis

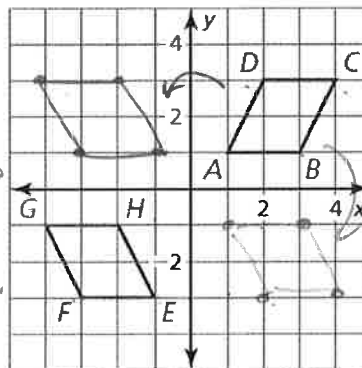
$\square BCDA \cong \square PQRN$
translation 6 units down and 2 units left



Example 1

Describe a congruence transformation that maps $\square ABCD$ to $\square EFGH$.

reflected over the y-axis
then translated down 4 units.



or #2

2. Describe another congruence transformation that maps $\square ABCD$ to $\square EFGH$.

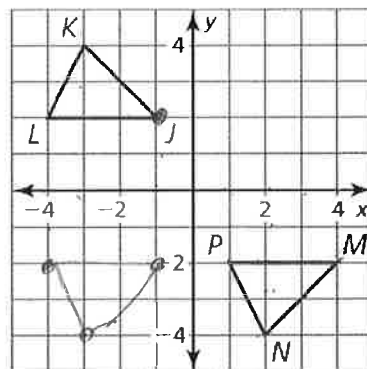
- reflected over the x-axis
then translated 5 units left.

Example 2

3. Describe a congruence transformation that maps $\triangle JKL$ to $\triangle MNP$.

$$\begin{aligned} J &(-1, 2) \\ K &(-3, 4) \\ L &(-4, 2) \end{aligned}$$

$$\begin{aligned} M &(4, 2) \\ N &(2, 4) \\ P &(1, -2) \end{aligned}$$



Step 1:
* reflect over
the x-axis

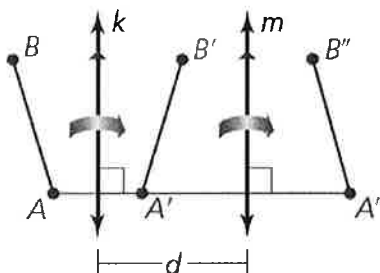
translate
5 units right

Monitoring Progress 2-3

Reflection in Parallel Lines Theorem:

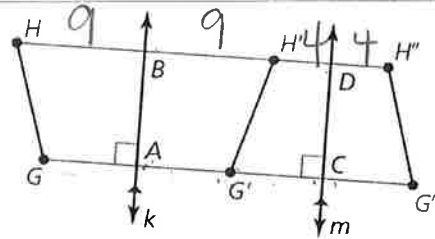
The composition of two reflections over parallel lines is equivalent to a translation.

- The length of the translation is twice the distance between the two lines.
- The translation vector is perpendicular to the two parallel lines.



Theorem

In the diagram, a reflection in line k maps \overline{GH} to $\overline{G'H'}$. A reflection in line m maps $\overline{G'H'}$ to $\overline{G''H''}$. Also, $HB = 9$ and $DH'' = 4$.



a. Name any segments congruent to each segment: \overline{GH} , \overline{HB} , and \overline{GA} .

$$\overline{GH} \cong \overline{G'H'} \cong \overline{G''H''} \quad \overline{HB} \cong \overline{BH'} \quad \overline{GA} \cong \overline{AG'}$$

b. Does $AC = BD$? Explain.

YES, $BDCA$ is a rectangle & opposite sides of a rectangle are equal.

c. What is the length of $\overline{GG''}$?

$$GG'' = HH'' = 2BD$$

$$26 = 2x$$

$$13 = x$$

$$BD = x$$

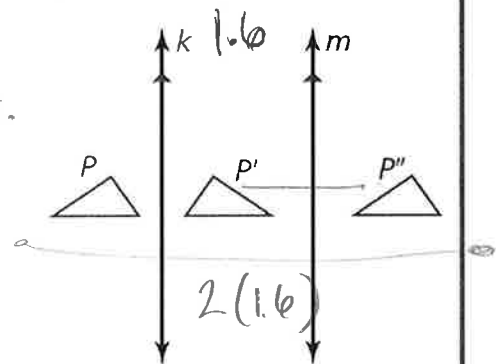
$$BD = 13$$

Example 3

Use the figure. The distance between line k and line m is 1.6 centimeters.

4. The preimage is reflected in line k , then in line m . Describe a single transformation that maps the blue figure to the green figure.

translation 3.2cm to the right.



5. What is the relationship between $\overline{PP'}$ and line k ? Explain.

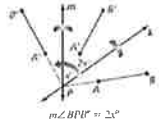
$\overline{PP'} \perp$ line k by Reflections in parallel lines.

6. What is the distance between P and P'' ?


3.2 cm.

Reflections in Intersecting Lines Theorem:
 Reflecting in two intersecting lines is the same as a rotation.

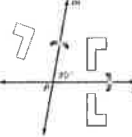
- The center of rotation is where the two lines intersect
- The angle of rotation is twice the angle formed by the intersecting lines.



In the diagram, the figure is reflected in line k . The image is then reflected in line m . Describe a single transformation that maps F to F' .



In the diagram, the preimage is reflected in line k , then in line m . Describe a single transformation that maps the blue figure onto the green figure.



8. A rotation of 76° maps C to C' . To map C to C' using two reflections, what is the measure of the angle formed by the intersecting lines of reflection?

→ a rotation of 116° about Point P.

→ 7.) 160° rotation about Point P.

→ 8.) 38°

Theorem

Homework

HW: pg. 204 #4-16, 19-22, 25-28

Closure