4.5 Dilations

$d = \sqrt{(x-x)^2+(y-y)^2}$

Determine whether $\overline{AB}$ and $\overline{CD}$ are congruent.

1. $A(-3, 4)$, $B(-3, 7)$, $C(3, -4)$, $D(93, -1)$

they are not $\cong$

$\sqrt{9} \neq \sqrt{8109}$

AB

$\sqrt{(-3+3)^2+(4-7)^2}$

$0^2+(-3)^2$

9

$AB = 3$

$\sqrt{(3-93)^2+(-4+1)^2}$

$(-90)^2+(-3)^2$

$8100+9$

8109

Find the midpoint of AB and CD.

2. $A(7, -2)$, $B(2, -2)$

$\left(\frac{7+2}{2}, \frac{-2-2}{2}\right)$

$(4.5, -2)$

3. $C(3, -4)$, $D(5, -4)$

$\left(\frac{3-5}{2}, \frac{-4+4}{2}\right)$

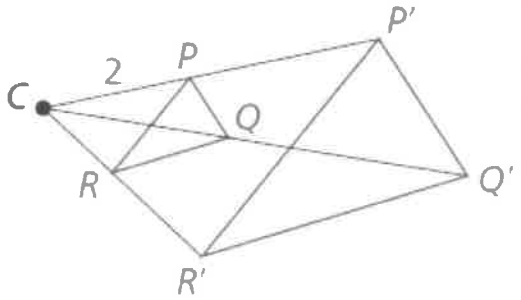
$(-1, 0)$

Dilations

A **dilation** is a transformation in which a figure is enlarged or reduced with respect to a fixed point C called the **center of dilation** and a **scale factor** k, which is the ratio of the lengths of the corresponding sides of the image and the preimage.

A dilation with center of dilation C and scale factor k maps every point P in a figure to a point P' so that the following are true.

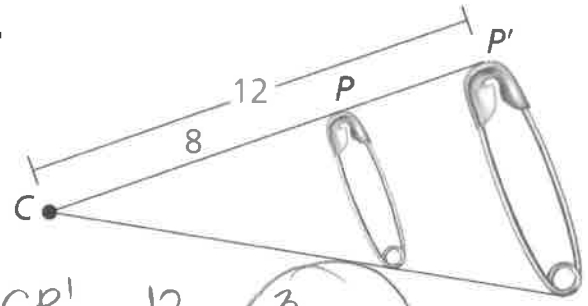
- If P is the center point C, then $P = P'$.
- If P is not the center point C, then the image point P' lies on $\overrightarrow{CP}$. The scale factor k is a positive number such that $k = \frac{CP'}{CP}$.
- Angle measures are preserved.



Core Concept

Find the scale factor of the dilation. Then tell whether the dilation is a *reduction* or an *enlargement*.

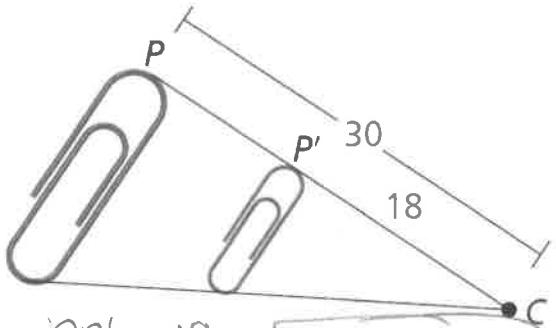
a.



$\frac{CP'}{CP} = \frac{12}{8} = \frac{3}{2} = k$

enlargement

b.



$\frac{CP'}{CP} = \frac{18}{30} = \frac{3}{5} = k$

reduction

c. In a dilation, $CP' = 3$ and $CP = 12$. Find the scale factor. Then tell whether the dilation is a *reduction* or an *enlargement*.

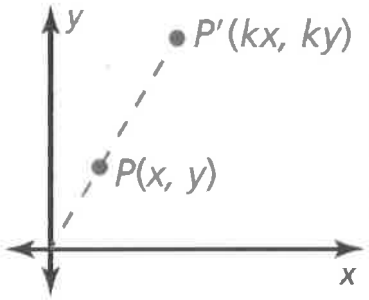
$\frac{CP'}{CP} = \frac{3}{12} = \frac{1}{4} = k$

reduction

Example 1

Coordinate Rule for Dilations

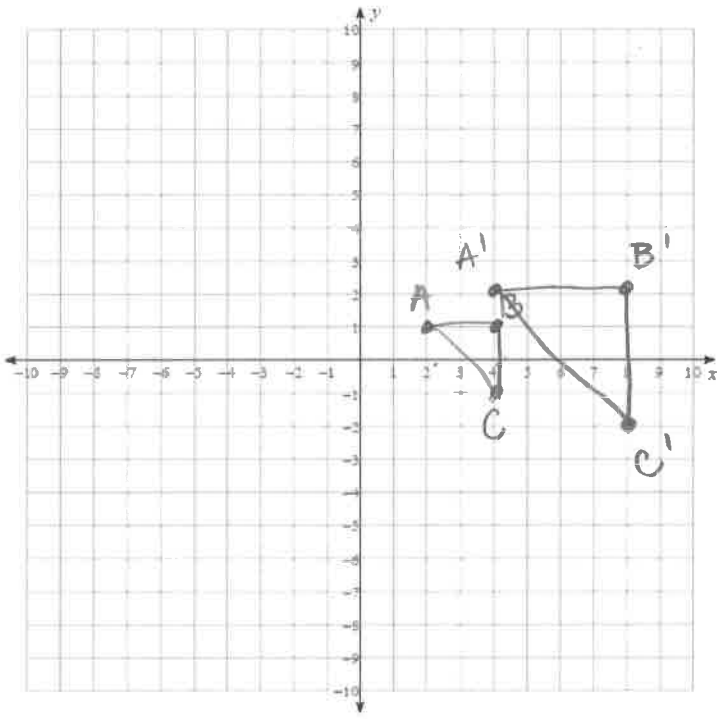
If $P(x, y)$ is the preimage of a point, then its image after a dilation centered at the origin (0, 0) with scale factor k is the point $P'(kx, ky)$.



$(x, y) \rightarrow (kx, ky)$

Core Concept

Graph △*ABC* with vertices *A*(2, 1), *B*(4, 1), and *C*(4, −1) and its image after a dilation with a scale factor of 2.

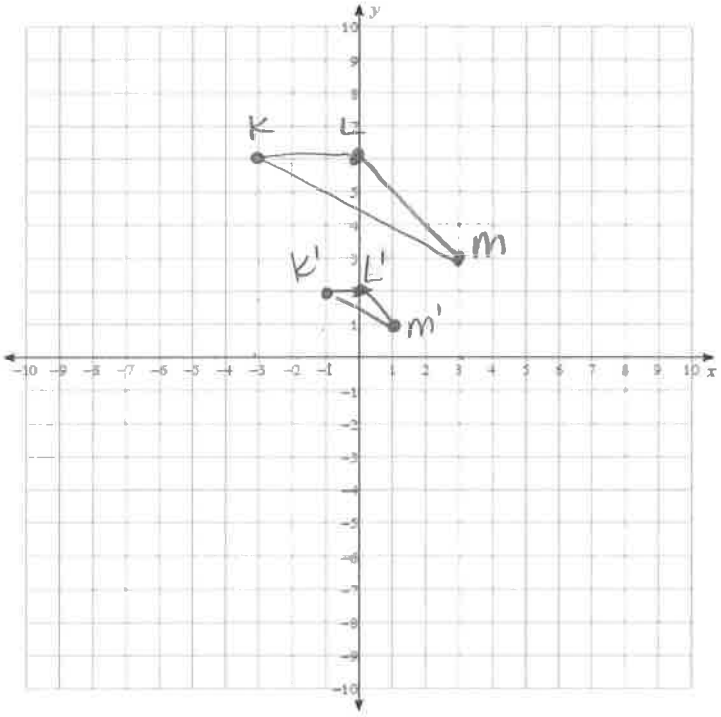


A'(4,2)
B'(8,2)
C'(8,-2)

Example 2

1/3

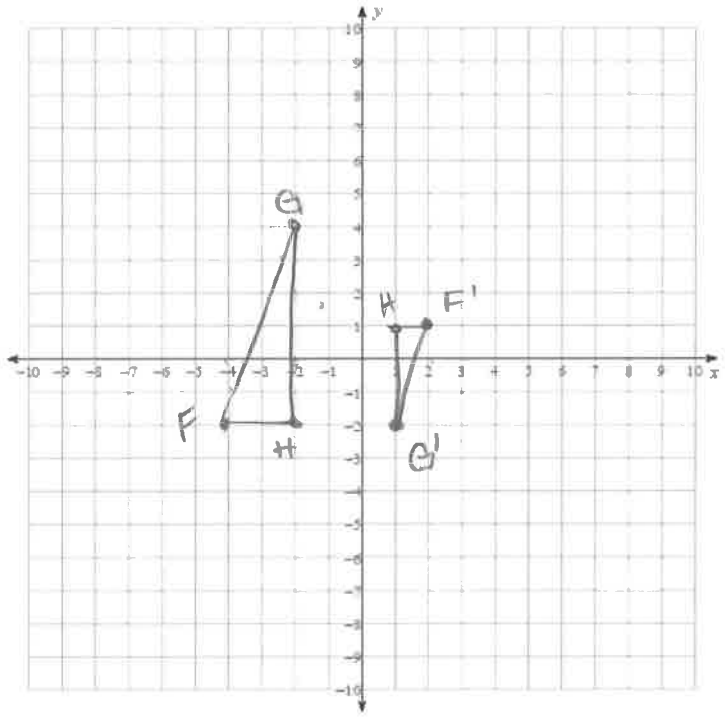
Graph quadrilateral *KLMN* with vertices *K*(−3, 6), *L*(0, 6), *M*(3, 3), and *N*(−3, −3) and its image after a dilation with a scale factor of $\frac{1}{3}$.



K'(−1, 2)
L'(0, 2)
m'(1, 1)

Example 3

Graph $\triangle FGH$ with vertices $F(-4, -2)$, $G(-2, 4)$, and $H(-2, -2)$ and its image after a dilation with a scale factor of $-\frac{1}{2}$.



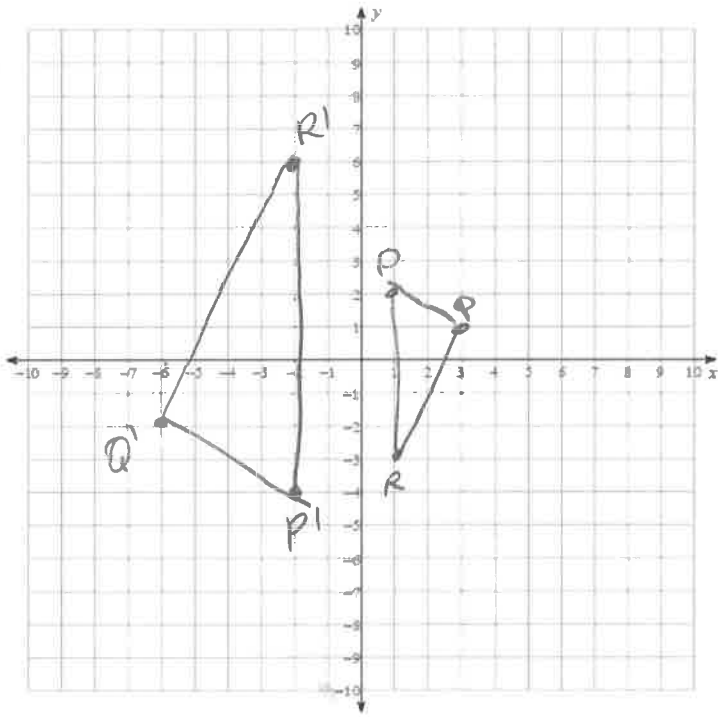
$F'(2,1)$

$G'(1,-2)$

$H'(1,1)$

Example 4

Find the coordinates image of $\triangle PQR$ with vertices $P(1, 2)$, $Q(3, 1)$, and $R(1, -3)$ after a dilation with a scale factor of -2.



$P'(-2, -4)$

$Q'(-6, -2)$

$R'(-2, 6)$

Suppose a figure containing the origin is dilated. Explain why the corresponding point in the image of the figure is also the origin.

– because

preimage $P(0,0) \rightarrow k = k \rightarrow$ image $P'(0 \cdot k, 0 \cdot k) = (0,0)$

$P'(0,0)$

You are making your own photo stickers. Your photo is 4 inches by 4 inches. The image on the stickers is 1.1 inches by 1.1 inches. What is the scale factor of this dilation?

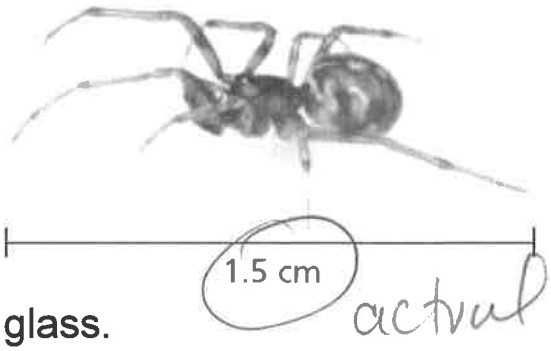
$\frac{\text{image length}}{\text{actual length}} = \frac{1.1}{4}$ (·10 / ·10) or $\frac{11}{40} = k$

An optometrist dilates the pupils of a patient's eyes to get a better look at the back of the eyes. A pupil dilates from 4.5 millimeters to 8 millimeters. What is the scale factor of this dilation?

$\frac{\text{image}}{\text{actual}} = \frac{8}{4.5}$ (·10 / ·10) $= \frac{80}{45} = k$

Example 5

You are using a magnifying glass that shows the image of an object that is six times the object's actual size. Determine the length of the image (x?) of the spider seen through the magnifying glass.



$$\frac{\text{image}}{\text{actual}} = \frac{6}{1} = \frac{x}{1.5}$$

$x = 9\text{ cm}$

Example 6

Homework:

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Nov 9-2:42 PM