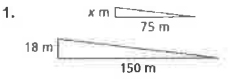
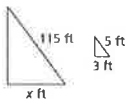
5.2 Congruent Triangles

Bellwork

The triangles are similar. Use proportions to find *x*.

1. 

$\frac{X}{18} = \frac{75}{150}$

2. 

$\frac{X}{3} = \frac{115}{5}$

$\frac{X}{3} = \frac{23}{1}$

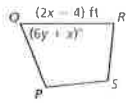
$X = 69$

$\frac{X}{18} = \frac{1}{2}$

$2X = 18$

$X = 9$

Naming Polygons



Helpful Hint

Two vertices that are the endpoints of a side are called consecutive vertices.

For example, *P* and *Q* are consecutive vertices.

To name a polygon, write the vertices in consecutive order. For example, you can name polygon *PQRS* as *QRSP* or *SRQP*, but **not** as *PRQS*.

In a congruence statement, pay attention to **the order of the vertices;** it indicates the corresponding parts.

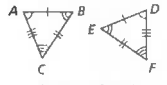
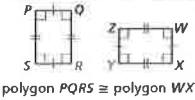
Warm Up

Geometric figures are congruent if they are the same size and shape.

Corresponding angles and **corresponding sides:** In the same position on a different figure of the same shape.

Corresponding parts of congruent triangles are congruent. (CPCTC)

Properties of Congruent Polygons

DIAGRAM	CORRESPONDING ANGLES	CORRESPONDING SIDES
 $\triangle ABC \cong \triangle DEF$	$\angle A \cong \angle D$ $\angle B \cong \angle E$ $\angle C \cong \angle F$	$\overline{AB} \cong \overline{DE}$ $\overline{BC} \cong \overline{EF}$ $\overline{AC} \cong \overline{DF}$
 polygon *PQRS* ≅ polygon *WXYZ*	$\angle P \cong \angle W$ $\angle Q \cong \angle X$ $\angle R \cong \angle Y$ $\angle S \cong \angle Z$	$\overline{PQ} \cong \overline{WX}$ $\overline{QR} \cong \overline{XY}$ $\overline{RS} \cong \overline{YZ}$ $\overline{PS} \cong \overline{WZ}$

Helpful Hint

When you write a statement such as $\Delta ABC \cong \Delta DEF$, you are also stating which parts are congruent.

Oct 22-1:44 PM

Given: $\Delta PQR \cong \Delta STW$

Identify all pairs of corresponding congruent parts.

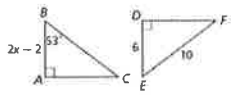
$\angle P \cong \angle S$ $\quad \overline{PQ} \cong \overline{ST}$

$\angle Q \cong \angle T$ $\quad \overline{QR} \cong \overline{TW}$

$\angle R \cong \angle W$ $\quad \overline{PR} \cong \overline{SW}$

Given: $\Delta ABC \cong \Delta DEF$

Find the value of x.



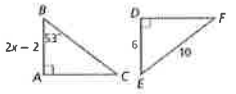
$2x - 2 = 6$

$2x = 8$

$x = 4$

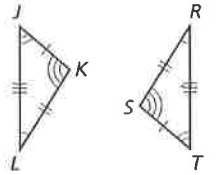
Given: $\Delta ABC \cong \Delta DEF$

Find $m\angle F$.



$m\angle F = m\angle C = 37°$

Write a congruence statement for the triangles. Identify all pairs of congruent corresponding parts.



$\triangle JKL \cong \triangle TSR$ (possible answer)

Nov 16-5:56 PM

In the diagram, $DEFG \cong SPQR$.

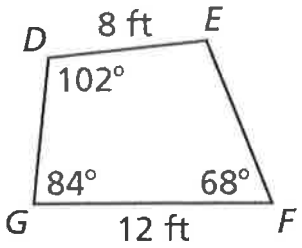
a. Find the value of x.

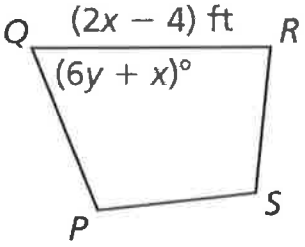
$QR = GF$

$2x - 4 = 12$

$2x = 16$

$x = 8$





b. Find the value of y.

$m\angle Q = m\angle F$

$6y + x = 68$

$6y + 8 = 68$

$6y = 60$

$y = 10$

Example 2

Theorem 5.3 Properties of Triangle Congruence

Triangle congruence is reflexive, symmetric, and transitive.

Reflexive For any triangle $\triangle ABC$, $\triangle ABC \cong \triangle ABC$.

Symmetric If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.

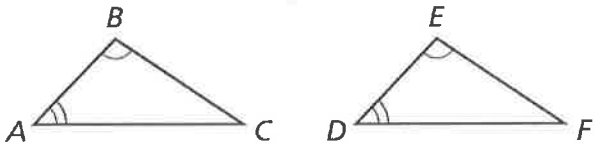
Transitive If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.

Proof *BigIdeasMath.com*

Theorem

Theorem 5.4 Third Angles Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.

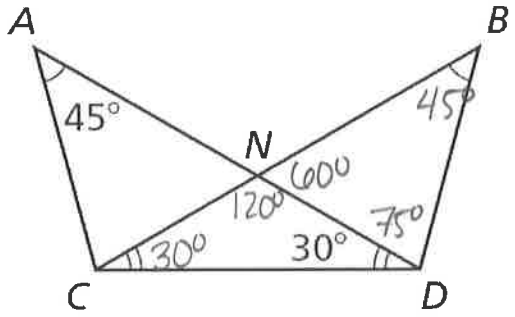


Proof Ex. 19, p. 244

If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\angle C \cong \angle F$.

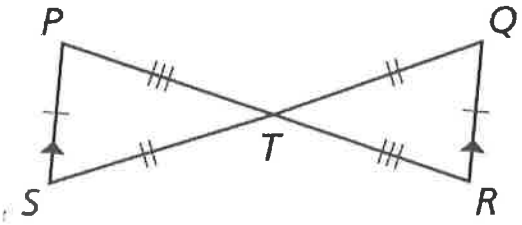
Theorem

Find $m\angle BDC$.



$m\angle BDC = 105°$

Example 4

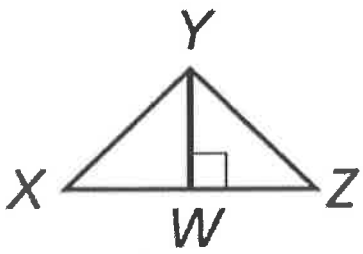


In the diagram at the left, Prove that $\triangle PTS \cong \triangle RTQ$.

S	R
$\overline{QR} \cong \overline{QR}$	Given
$\overline{QT} \cong \overline{ST}$	Given
$\overline{PT} \cong \overline{TR}$	Given
$\angle PTS \cong \angle QTR$	Vert. $\angle$ Theorem
$\overline{QR} \parallel \overline{PS}$	Given
$\angle S \cong \angle Q$	Alt. Int $\angle$'s Thm
$\angle P \cong \angle R$	Alt. Int $\angle$'s Thm
$\triangle PTS \cong \triangle RTQ$	Def. of $\cong$ $\triangle$'s

Monitoring Progress 3

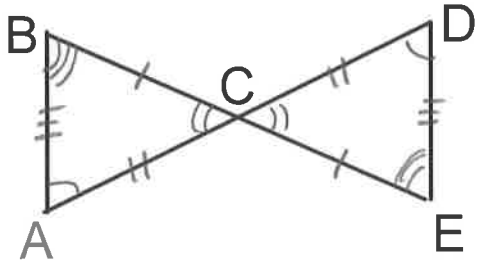
Given: $\angle YWX$ and $\angle YWZ$ are right angles.
$\overline{YW}$ bisects $\angle XYZ$. W is the midpoint of $\overline{XZ}$. $\overline{XY} \cong \overline{YZ}$.
Prove: $\Delta XYW \cong \Delta ZYW$



S	R
$\angle YWX$ and $\angle YWZ$ are right $\angle$'s	Given
$\angle YWX \cong \angle YWZ$	Right $\angle \cong$ Thm
$\overline{YW}$ bisects $\angle XYZ$	Given
$\angle XYW \cong \angle ZYW$	Def. of bisect
$\angle X \cong \angle Z$	Third $\angle$'s Thm
W is midpoint of $\overline{XZ}$	Given
$\overline{XW} \cong \overline{WZ}$	Def. of mdpt
$\overline{XY} \cong \overline{YZ}$	Given
$\overline{YW} \cong \overline{YW}$	Reflexive POC
$\Delta XYW \cong \Delta ZYW$	Def. of $\cong \Delta$'s

Nov 16-6:00 PM

Given: $\overline{AD}$ bisects $\overline{BE}$.
$\overline{BE}$ bisects $\overline{AD}$.
$\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$
Prove: $\Delta ABC \cong \Delta DEC$

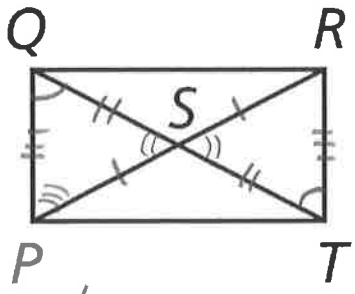


S	R
$\overline{AD}$ bisects $\overline{BE}$, $\overline{BE}$ bisects $\overline{AD}$	Given
$\overline{BC} \cong \overline{CE}$, $\overline{AC} \cong \overline{CD}$	Def. of bisect
$\overline{AB} \cong \overline{DE}$	Given
$\angle A \cong \angle D$	Given
$\angle BCA \cong \angle ECD$	Vert. $\angle$'s Thm
$\angle B \cong \angle E$	Third $\angle$'s Thm
$\Delta ABC \cong \Delta DEC$	Def. of $\cong \Delta$'s

Nov 16-6:01 PM

Given: *PR* and *QT* bisect each other.
$\angle PQS \cong \angle RTS$, $QP \cong RT$

Prove: $\Delta QPS \cong \Delta TRS$

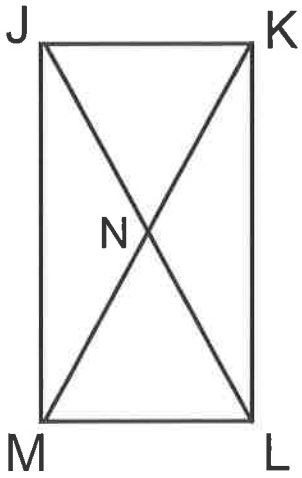


1.	1. Given
2. PR & QT bisect each other	2. Def. of bisect
3. $\overline{PS} \cong \overline{SR}$, $\overline{QS} \cong \overline{ST}$	3. Given
4. $\overline{QP} \cong \overline{RT}$	4. Given
5. $\angle PQS \cong \angle RTS$	5. Vert. $\angle$'s Thm
6. $\angle QSP \cong \angle TSR$	6. Third $\angle$'s Thm
7. $\angle QPS \cong \angle SRT$	7. Third $\angle$'s Thm
8. $\Delta QPS \cong \Delta TRS$	8. Def. of $\cong$ Δ's

Nov 16-6:01 PM

Given: $\overline{MK}$ bisects $\overline{JL}$. $\overline{JL}$ bisects $\overline{MK}$. $\overline{JK} \cong \overline{ML}$.
$\overline{JK} \parallel \overline{ML}$.

Prove: $\Delta JKN \cong \Delta LMN$



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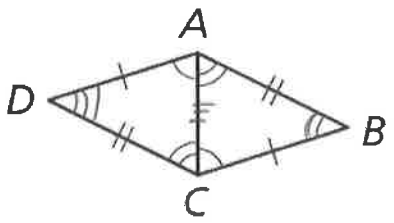
Use the information in the figure to prove that $\triangle ACD \cong \triangle CAB$.



S	R
$\overline{AD} \cong \overline{BC}$, $\overline{AB} \cong \overline{CD}$	Given
$\overline{AC} \cong \overline{AC}$	Reflexive POC
$\angle DAC \cong \angle BCA$	Given
$\angle ACD \cong \angle CAB$	Given
$\angle D \cong \angle B$	Third $\angle$'s Thm
$\triangle ACD \cong \triangle CAB$	Def. of $\cong$ $\triangle$'s

Example 5

Homework:

pg. 243 #3-10, 13,14, 24

Finish 5.2 Proof WS

Closure