

5.4 Equilateral and Isosceles Triangles

Rewrite the definition of the term as a biconditional statement.

1. In an isosceles triangle, the legs are of equal length.

A Δ is isosceles iff the legs are equal length.

2. A tangram is a Chinese puzzle made up of seven pieces.

A Chinese puzzle is made up of 7 pieces iff it is a tangram.

3. A rectangle is a parallelogram that has four right angles.

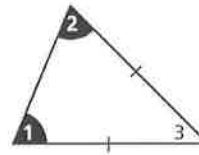
A figure is a rectangle iff it is a p-gram with 4 right \angle 's.

Cumulative Warm Up

Recall that an isosceles triangle has at least two congruent sides. The congruent sides are called the **legs**. The **vertex angle** is the angle formed by the legs. The side opposite the vertex angle is called the **base**, and the **base angles** are the two angles that have the base as a side.

$\angle 3$ is the vertex angle.

$\angle 1$ and $\angle 2$ are the base angles.



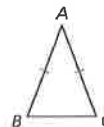
Theorems

Theorem 5.6 Base Angles Theorem

If two sides of a triangle are congruent, then the angles opposite them are congruent.

If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong \angle C$.

Proof p. 252

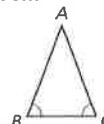


Theorem 5.7 Converse of the Base Angles Theorem

If two angles of a triangle are congruent, then the sides opposite them are congruent.

If $\angle B \cong \angle C$, then $\overline{AB} \cong \overline{AC}$.

Proof Ex. 27, p. 275



$$180 - 22 = 158$$

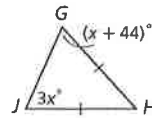
$$\frac{158}{2} = 79^\circ$$

$$m\angle F = 79^\circ$$

Find $m\angle F$.



Find $m\angle G$.



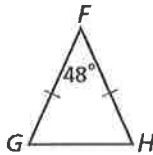
$$\begin{array}{r} x + 44 = 3x \\ -x \quad -x \\ \hline 44 = 2x \end{array}$$

$$\frac{44}{2} = \frac{2x}{2}$$

$$22 = x$$

$$m\angle G = 22 + 44 = 66^\circ$$

Find $m\angle H$.

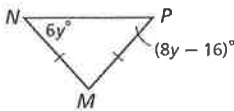


$$m\angle H = 66^\circ$$

$$180 - 48 = 132^\circ$$

$$\frac{132^\circ}{2} = 66^\circ$$

Find $m\angle N$.



$$6y = 8y - 16$$

$$\begin{array}{r} 6y - 8y = -16 \\ -2y = -16 \\ \hline -2 \quad -2 \\ \hline y = 8 \end{array}$$

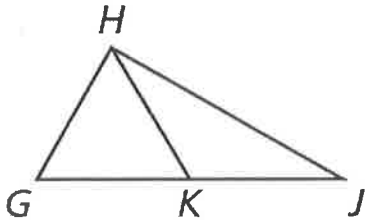
$$\begin{array}{l} m\angle N = 6(8) \\ m\angle N = 48^\circ \end{array}$$

Nov 13-1:43 PM

Copy and complete the statement.

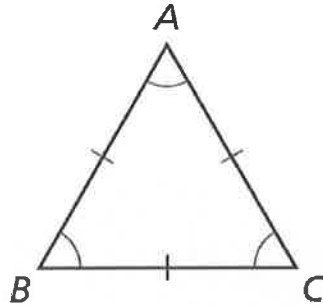
1. If $\overline{HG} \cong \overline{HK}$, then $\angle \underline{G} \cong \angle \underline{GKH}$.

2. If $\angle KHJ \cong \angle KJH$, then $\underline{HK} \cong \underline{JK}$.



If a triangle is equilateral, then it is equiangular.

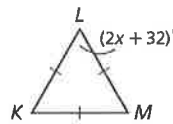
If a triangle is equiangular, then it is equilateral.



Corollary

Find the value of x .

$\triangle LKM$ is equilateral.



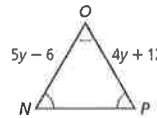
$$\begin{array}{r} 2x + 32 = 60 \\ -32 \quad -32 \\ \hline \end{array}$$

$$\frac{2x}{2} = \frac{28}{2}$$

$$x = 14$$

Find the value of y .

$\triangle NPO$ is equiangular.

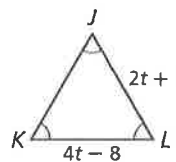


$$\begin{array}{r} 5y - 6 = 4y + 12 \\ -4y + 6 \quad -4y + 6 \\ \hline \end{array}$$

$$y = 18$$

Find the value of JL .

$\triangle JKL$ is equiangular.



$$JL = 2(4.5) + 1$$

$$JL = 10$$

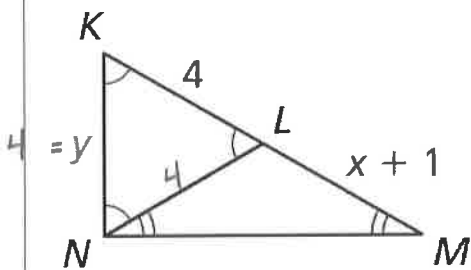
$$\begin{array}{r} 2t + 1 = 4t - 8 \\ -2t + 8 \quad -2t + 8 \\ \hline \end{array}$$

$$\frac{9}{2} = \frac{2t}{2}$$

$$4.5 = t$$

Example 2

Find the values of x and y in the diagram.



$$x + 1 = 4$$

$$x = 3$$

$$y = 4$$

Example 3

S

$\overline{PS} \cong \overline{QR}$
 $\angle QPS \cong \angle PQR$
 $\overline{PQ} \cong \overline{PQ}$
 $\triangle QPS \cong \triangle PQR$

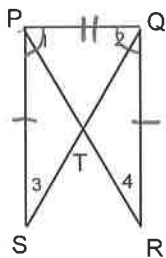
R

Given
 Given
 Reflexive
 SAS

$\overline{PS} \cong \overline{QR}$ and $\angle QPS \cong \angle PQR$.

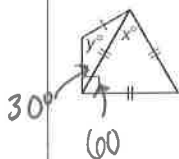
a. Prove: $\triangle QPS \cong \triangle PQR$.

b. Explain why $\triangle PQT$ is isosceles.



$\angle 3 \cong \angle 4$ by CPCTC
 $\angle PTS \cong \angle QTR$ by Vert. \angle 's Thm
 $\triangle PTS \cong \triangle QTR$ by AAS
 $\overline{PT} \cong \overline{QT}$ by CPCTC
 $\triangle PQT$ is isosceles - Def. of Isosceles

4. Find the values of x and y in the diagram.



$$x = 60^\circ ; y = 120^\circ$$

$$180 - 30 - 30 = 120^\circ$$

Example 4

Homework

Pg. 256 #4-10 Evens, 13-16, 22-24, 29, 30

Closure