5.7 Using Congruent Triangles

Name the property the statement illustrates.

1. If $\overline{RU} \cong \overline{WX}$ and $\overline{WX} \cong \overline{YZ}$, then $\overline{RU} \cong \overline{YZ}$.

Transitive Property of Congruence

2. $\angle A \cong \angle A$

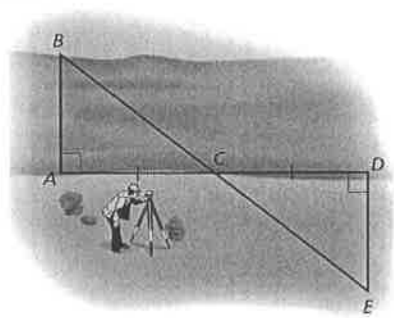
Reflexive Prop. of $\cong$

3. If $\angle B \cong \angle C$, then $\angle C \cong \angle B$.

Symmetric Prop. of $\cong$

Warm Up 1-3

The figure shows how a surveyor can measure the width of a river by making measurements on only one side of the river.



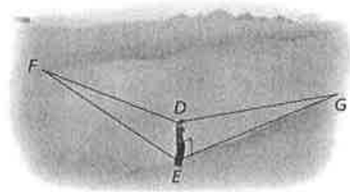
Write a proof to verify that the method you described in part (a) is valid.

Given $\angle A$ is a right angle, $\angle D$ is a right angle, $\overline{AC} \cong \overline{CD}$

1. $\overline{AC} \cong \overline{CD}$, $\angle A$ & $\angle D$ are rt. $\angle$'s	1. given
2. $\angle A \cong \angle D$	2. Rt. $\angle$'s $\cong$ Thrm.
3. $\angle ACB \cong \angle DCE$	3. Vert. $\angle$'s $\cong$ Thrm.
4. $\triangle ABC \cong \triangle DEC$	4. ASA $\cong$ Thrm.
5. $\overline{AB} \cong \overline{DE}$	5. CPCTC
6. $AB = DE$	6. Def. of $\cong$ segments.

Exploration 1a-b

It was reported that one of Napoleon's officers estimated the width of a river as follows. The officer stood on the bank of the river and lowered the visor on his cap until the farthest thing visible was the edge of the bank on the other side. He then turned and noted the point on his side that was in line with the tip of his visor and his eye. The officer then paced the distance to this point and concluded that distance was the width of the river.



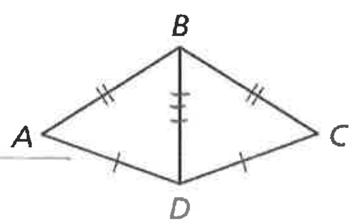
Write a proof to verify that the conclusion the officer made is correct.

1. **Given** $\angle DEG$ is a right angle, $\angle DEF$ is a right angle, $\angle EDG \cong \angle EDF$ — 1. given
2. $\angle DEG \cong \angle DEF$ — 2. Rt. $\angle$'s $\cong$ Thrm.
3. $\overline{DE} \cong \overline{DE}$ — 3. Reflexive Prop. of $\cong$
4. $\triangle DEF \cong \triangle DEF$ — 4. ASA $\cong$ Thrm.
5. $\overline{EG} \cong \overline{EF}$ — 5. CPCTC
6. $EG = EF$ — 6. Def. of $\cong$ segments

Exploration 2

Given: $\overline{AB} \cong \overline{BC}$, $\overline{AD} \cong \overline{DC}$

Prove: $\angle A \cong \angle C$



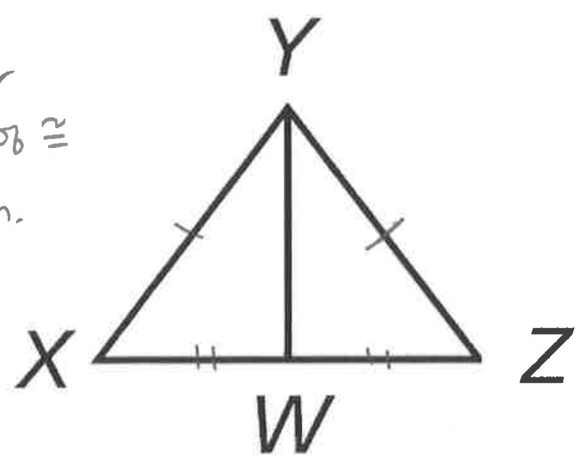
1. $\overline{AB} \cong \overline{BC}$; $\overline{AD} \cong \overline{DC}$ — 1. given
2. $\overline{BD} \cong \overline{BD}$ — 2. Reflexive Prop. of $\cong$
3. $\triangle ABD \cong \triangle CBD$ — 3. SSS $\cong$ Thrm.
4. $\angle A \cong \angle C$ — 4. CPCTC

Monitoring Progress 1

Given: $\overline{YW}$ bisects $\overline{XZ}$, $\overline{XY} \cong \overline{YZ}$.

Prove: $\angle XYW \cong \angle ZYW$

1. $\overline{XY} \cong \overline{YZ}$; $\overline{YW}$ bisects $\overline{XZ}$	1. given
2. $\overline{XW} \cong \overline{ZW}$	2. def. of bisector
3. $\overline{YW} \cong \overline{YW}$	3. Reflexive Prop. of $\cong$
4. $\triangle XYW \cong \triangle ZYW$	4. SSS $\cong$ Thrm.
5. $\angle XYW \cong \angle ZYW$	5. CPCTC

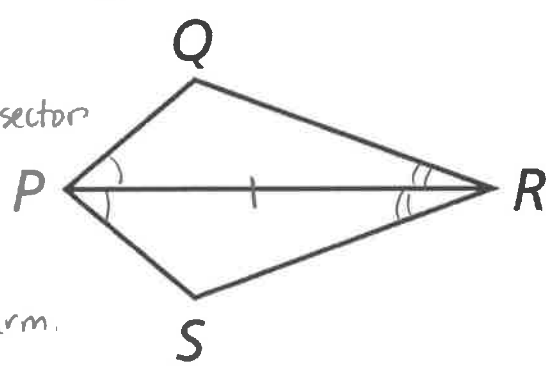


Nov 20-5:25 PM

Given: $\overline{PR}$ bisects $\angle QPS$ and $\angle QRS$.

Prove: $\overline{PQ} \cong \overline{PS}$

1. $\overline{PR}$ bisects $\angle QPS$ and $\angle QRS$	1. given
2. $\angle QPR \cong \angle SPR$ and $\angle QRP \cong \angle SRP$	2. def. of $\angle$ bisector
3. $\overline{PR} \cong \overline{PR}$	3. Reflexive prop $\cong$
4. $\triangle QPR \cong \triangle SPR$	4. ASA $\cong$ Thrm.
5. $\overline{PQ} \cong \overline{PS}$	5. CPCTC



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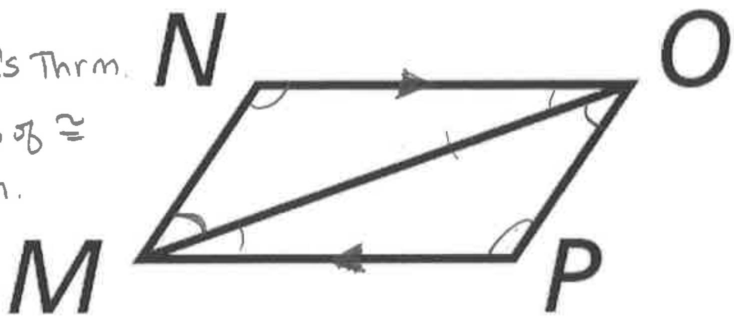
Given: $\overline{NO} \parallel \overline{MP}$, $\angle N \cong \angle P$

Prove: $\overline{MN} \parallel \overline{OP}$

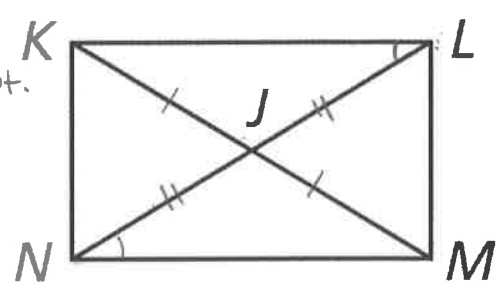


1. $\overline{NO} \parallel \overline{MP}$, $\angle N \cong \angle P$	1. given
2. $\angle MOP \cong \angle OMN$	2. Alt. Int. ∠'s Thrm.
3. $\overline{MO} \cong \overline{MO}$	3. Reflex. Prop of ≅
4. $\triangle POM \cong \triangle NMO$	4. AAS Thrm.
5. $\angle NOM \cong \angle PMO$	5. CPCTC
6. $\overline{MN} \parallel \overline{OP}$	6. Conv. Alt. Int. ∠'s Thrm.

Nov 29-9:32 AM

Given: J is the midpoint of $\overline{KM}$ and $\overline{NL}$.

Prove: $\overline{KL} \parallel \overline{MN}$

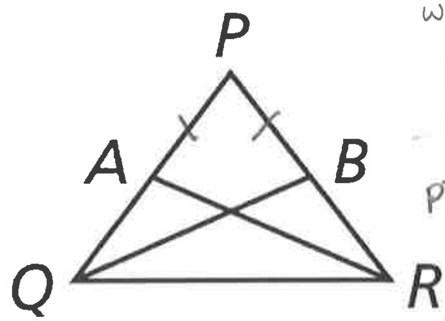


1. J is mdpt. of $\overline{KM}$ & $\overline{NL}$	1. given
2. $\overline{KJ} \cong \overline{MJ}$, $\overline{NJ} \cong \overline{LJ}$	2. def of mdpt.
3. $\angle KJN \cong \angle MJL$	3. def of Vert. ∠'s
4. $\triangle KJN \cong \triangle MJL$	4. SAS ≅ Thrm.
5. $\angle KLJ \cong \angle MNL$	5. CPCTC
6. $\overline{KL} \parallel \overline{MN}$	6. Conv. Alt. Int. ∠'s Thrm.

Nov 29-9:33 AM

1. Given: Isosceles ΔPQR, base QR, $\overline{PA} \cong \overline{PB}$

Prove: $\overline{AR} \cong \overline{BQ}$



S	R
Isosceles ΔPQR w/ base QR	Given
$\overline{PA} \cong \overline{PB}$	Given
$\overline{PR} \cong \overline{PQ}$	Def. of Isosceles
$\angle P \cong \angle P$	Reflexive POC
$\Delta PAR \cong \Delta PBQ$	SAS
$\overline{AR} \cong \overline{BQ}$	CPCTC

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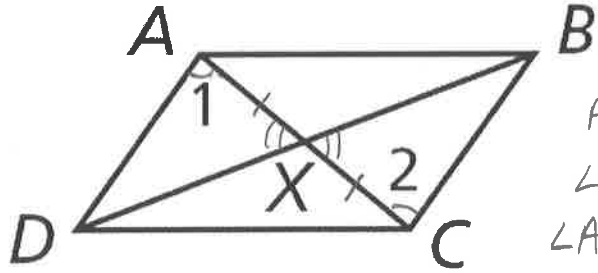
2. Given: X is the midpoint of AC . $\angle 1 \cong \angle 2$

Prove: X is the midpoint of BD.

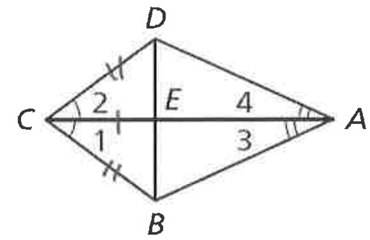


S	R
X mdpt of AC	Given
$\overline{AX} \cong \overline{XC}$	Def of mdpt
$\angle 1 \cong \angle 2$	Given
$\angle AXD \cong \angle CXB$	Vert. $\angle$'s $\cong$ Theorem
$\Delta AXD \cong \Delta CXB$	ASA
$\overline{BX} \cong \overline{XD}$	CPCTC
X mdpt of $\overline{BD}$	Def. of mdpt

Nov 29-9:33 AM

Given $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

Prove $\triangle BCE \cong \triangle DCE$



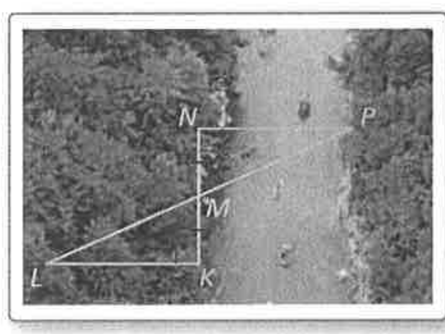
S	R
$\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	Given
$\overline{AC} \cong \overline{AC}$	Reflexive POC
$\triangle ACD \cong \triangle ACB$	ASA
$\overline{CD} \cong \overline{CB}$	CPCTC
$\overline{CE} \cong \overline{CE}$	Reflexive POC
$\triangle BCE \cong \triangle DCE$	SAS

Example 3

Use the following method to find the distance across a river, from point N to point P.

- Place a stake at K on the near side so that $\overline{NK} \perp \overline{NP}$.
- Find M, the midpoint of $\overline{NK}$.
- Locate the point L so that $\overline{NK} \perp \overline{KL}$ and L, P, and M are collinear.

Explain how this plan allows you to find the distance.



$\angle N$ and $\angle K$ are both right $\angle$'s by def of $\perp$

$\angle N \cong \angle K$ by Right $\angle \cong$ Thm

$\overline{NM} \cong \overline{MK}$ by Def. of midpoint

$\angle NMP \cong \angle KML$ by Vert. $\angle$'s Thm

$\triangle PNM \cong \triangle LKM$ by ASA

Therefore $\overline{LK} \cong \overline{NP}$ by CPCTC

You can find the length of LK which will give you the length of NP

Example 2

HW: WS Proofs Involving Congruent Triangles and CPCTC

Closure