6.1 Perpendicular and Angle Bisectors

mdpt

$\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

$\left(\frac{2+(-4)}{2}, \frac{8+6}{2}\right)$

$(-1, 7)$

Bellwork

1. Find the midpoint of the segment (2, 8) and (−4, 6)

2. Find the slope of the segment (2, 8) and (−4, 6)

$-6\left(\begin{matrix}2, 8\\ -4, 6\end{matrix}\right)-2$

$m = \frac{-2}{-6} = \frac{1}{3}$

Objectives

Prove and apply theorems about perpendicular bisectors.

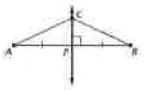
Equidistant: When a point is the same distance from two or more objects

Theorem 6.1 Perpendicular Bisector Theorem

In a plane, if a point lies on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

If $\overleftrightarrow{CP}$ is the ⊥ bisector of $\overline{AB}$, then $CA = CB$.

Proof p. 302

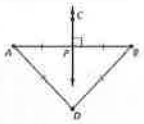


Theorem 6.2 Converse of the Perpendicular Bisector Theorem

In a plane, if a point is equidistant from the endpoints of a segment, then it lies on the perpendicular bisector of the segment.

If $DA = DB$, then point D lies on the ⊥ bisector of $\overline{AB}$.

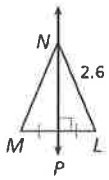
Proof Ex. 32, p. 308



Find each measure.

MN

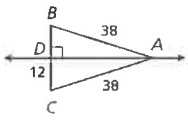
2.6



Find each measure.

BC

24



Find each measure.

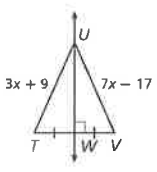
TU

$3x + 9 = 7x - 17$

$-3x + 17 \quad -3x + 17$

$\frac{26}{4} = \frac{4x}{4}$

$6.5 = x$



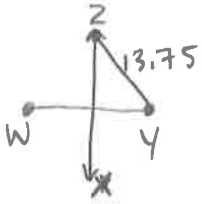
$TU = 3(6.5) + 9$

$= 19.5 + 9$

$TU = 28.5$

Use the diagram and the given information to find the indicated measure.

1. $\overleftrightarrow{ZX}$ is the perpendicular bisector of $\overline{WY}$, and $YZ = 13.75$. Find WZ.



$WZ = 13.75$

2. $\overleftrightarrow{ZX}$ is the perpendicular bisector of $\overline{WY}$, $WZ = 4n - 13$, and $YZ = n + 17$. Find YZ.

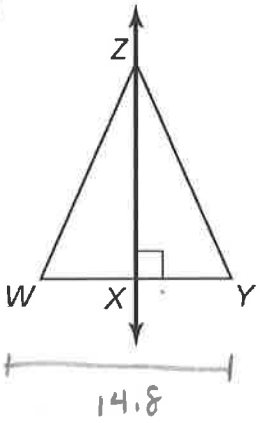
$4n - 13 = n + 17$

$3n = 30$

$n = 10$

$YZ = 10 + 17$

$YZ = 27$



3. Find WX when $WZ = 20.5$, $WY = 14.8$, and $YZ = 20.5$.

$WX = \frac{14.8}{2} = 7.4$

Write an equation in point-slope form for the perpendicular bisector of the segment with endpoints $C(6, -5)$ and $D(10, 1)$.

mdpt:

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\left(\frac{6+10}{2}, \frac{-5+1}{2}\right)$

$(8, -2)$

slope $= +4\left(\begin{matrix}6, -5\\10, 1\end{matrix}\right)+6$

$m = \frac{6}{4} = \frac{3}{2}$

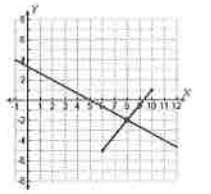
$\perp m = -\frac{2}{3}$

$y = mx + b$

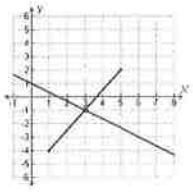
$-2 = 8\left(-\frac{2}{3}\right) + b$

$-\frac{6}{3} = -\frac{16}{3} + b \quad b = \frac{10}{3}$

$y = -\frac{2}{3}x + \frac{10}{3}$



Write an equation in point-slope form for the perpendicular bisector of the segment with endpoints $P(5, 2)$ and $Q(1, -4)$.



mdpt

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\left(\frac{5+1}{2}, \frac{2+(-4)}{2}\right)$

$(3, -1)$

$-4\left(\begin{matrix}5, 2\\1, -4\end{matrix}\right)-6 \quad m = \frac{-6}{-4} = \frac{3}{2}$

$\perp m = -\frac{2}{3}$

$y = mx + b$

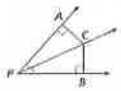
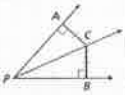
$-1 = -\frac{2}{3}(3) + b$

$-1 = -2 + b$

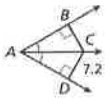
$1 = b$

$y = -\frac{2}{3}x + 1$

Theorems Distance and Angle Bisectors

THEOREM	HYPOTHESIS	CONCLUSION
Angle Bisector Theorem If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.	 $\angle APC \cong \angle BPC$	$AC = BC$
Converse of the Angle Bisector Theorem If a point in the interior of an angle is equidistant from the sides of the angle, then it is on the bisector of the angle.	 $AC = BC$	$\angle APC \cong \angle BPC$

Find the measure.
BC

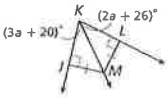


$BC = 7.2$

Find the measure.
m∠*EFH*, given that m∠*EFG* = 50°.

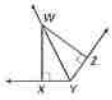
$m\angle EFH = 25°$

Find m∠*MKL*.



$3a + 20 = 2a + 26$
$-2a \quad -20 \quad -2a \quad -20$
$a = 6$
$m\angle LKM = 2(6) + 26 = 42°$

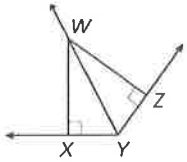
Given that *YW* bisects ∠*XYZ* and *WZ* = 3.05, find *WX*.



$WX = 3.05$

Given that m∠*WYZ* = 63°, *XW* = 5.7, and *ZW* = 5.7, find m∠*XYZ*.

$m\angle XYZ = 2(63) = 126°$



Homework:

pg. 307 # 4-20, 24,29,30