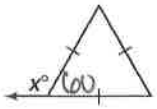
Chapter 7 - Polygons and Quadrilaterals

7.1 Angles in Polygons

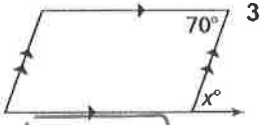
Bellwork

Find the value of *x* in the diagram.

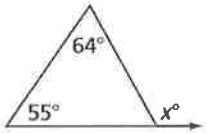
1. 

X = 180 - 60

X = 120

2. 

X = 70

3. 

X = 55 + 64

X = 119

Objectives

Classify polygons based on their sides and angles.

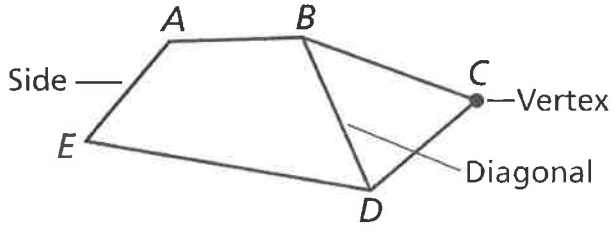
Find and use the measures of interior and exterior angles of polygons.

A polygon is a closed plane figure formed by three or more segments that intersect only at their endpoints.

side of the polygon - segments that forms a polygon

vertex of the polygon - common endpoint of two sides

Diagonal - A segment that connects any two *nonconsecutive* vertices



You can name a polygon by the number of its sides.

Number of Sides	Name of Polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon
n	*n*-gon

11 sides: undecagon or hendecagon
13 sides: tridecagon
14 sides: tetradecagon
15 sides: pentadecagon
16 sides: hexadecagon
17 sides: heptadecagon
18 sides: octadegagon
19 sides: enneadecagon
20 sides: icosagon

A polygon is **concave** if any part of a diagonal contains points in the exterior of the polygon.

(Concave – part of the shape is CAVED IN.)

Concave quadrilateral

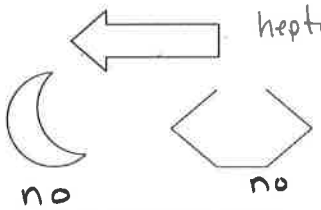
If no diagonal contains points in the exterior, then the polygon is **convex**.

Convex quadrilateral

A **regular polygon** is one that is both equilateral and equiangular. A regular polygon is always convex.

If a polygon is not regular, it is called irregular.

Tell whether the figure is a polygon. If it is a polygon, name it by the number of sides.

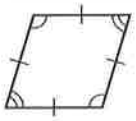


heptagon (concave)

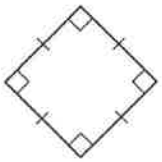
no

no

Tell whether the polygon is regular or irregular. Tell whether it is concave or convex.

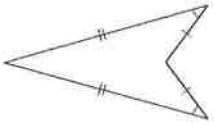


convex, irregular



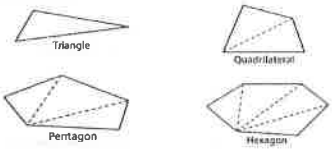
convex, regular

convex, regular



concave, irregular

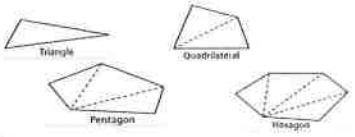
To find the sum of the interior angle measures of a convex polygon, draw all possible diagonals from one vertex of the polygon. This creates a set of triangles. The sum of the angle measures of all the triangles equals the sum of the angle measures of the polygon.



Remember!

By the Triangle Sum Theorem, the sum of the interior angle measures of a triangle is 180°.

Polygon	Number of Sides	Number of Triangles	Sum of Interior Angle Measures
Triangle	3	1	(1)180° = 180°
Quadrilateral	4	2	(2)180° = 360°
Pentagon	5	3	(3)180° = 540°
Hexagon	6	4	(4)180° = 720°
n-gon	n	$n-2$	$(n-2)180°$

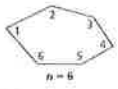


Theorem 7.1 Polygon Interior Angles Theorem

The sum of the measures of the interior angles of a convex n-gon is $(n-2) \cdot 180°$.

$$m\angle 1 + m\angle 2 + \cdots + m\angle n = (n-2) \cdot 180°$$

Proof Ex. 42 (for pentagons), p. 365



Corollary

Corollary 7.1 Corollary to the Polygon Interior Angles Theorem

The sum of the measures of the interior angles of a quadrilateral is 360°.

Proof Ex. 43, p. 366

$(n-2)180$

Find the sum of the interior angle measures of a convex heptagon.

$(7-2)180$

$(5)180 = \boxed{900°}$

Find the measure of each interior angle of a regular 16-gon.

$(16-2)180$

$(14)180 = 2520$

$\frac{2520}{16} = \boxed{157.5°}$

The sum of the measures of the interior angles of a convex polygon is 900°. Classify the polygon by the number of sides.

$\frac{(n-2)180}{180} = \frac{900}{180} \rightarrow n-2=5$

$+2 \quad +2$

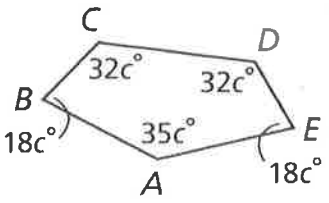
$\boxed{n=7}$

Find the measure of angle A in polygon ABCDE

$2(18c) + 2(32c) + 35c = (5-2)180$

$\frac{135c}{135} = \frac{540}{135}$

$c = 4$



$mLA = 35(4)$

$\boxed{mLA = 140°}$

Find the sum of the interior angle measures of a convex 15-gon.

$(15-2)180$

$(13)180$

2340°

Find the measure of each interior angle of a regular decagon.

$(10-2)180$

$(8)180$

1440

$\frac{1440}{10} = 144^\circ$

The sum of the measures of the interior angles of a convex polygon is 1440°. Classify the polygon by the number of sides.

$\frac{(n-2)180}{180} = \frac{1440}{180}$

$n-2=8$

$+2 \quad +2$

$n=10$

The measures of the interior angles of a quadrilateral are $x°$, $3x°$, $5x°$, and $7x°$. Find the measures of all the interior angles.

$x + 3x + 5x + 7x = 360$

$16x = 360$

$x = 22.5$

$x = 22.5$

$3x = 67.5$

$5x = 112.5$

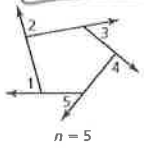
$7x = 157.5$

Theorem 7.2 Polygon Exterior Angles Theorem

The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360°.

$m\angle 1 + m\angle 2 + \cdots + m\angle n = 360°$

Proof Ex. 51, p. 366



$n = 5$

Remember!

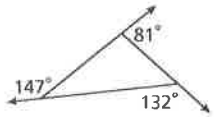
An exterior angle is formed by one side of a polygon and the extension of a consecutive side.

Proof: Because each of the n interior angles forms a linear pair with its corresponding exterior angle, you know that the sum of the measures of the n interior angles is 180n.

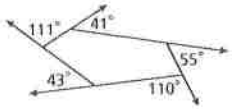
Subtracting the sum of the interior angle measures from the sum of the measures of the linear pairs gives you:

$180n - [(n - 2)180] = 360$

Examples



$147° + 81° + 132° = 360°$

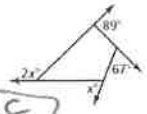


$43° + 111° + 41° + 55° + 110° = 360°$

Find the measure of each exterior angle of a regular 20-gon.

$\frac{360}{20} = 18^\circ$

Find the value of x in the diagram.



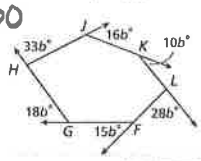
$X + 2X + 89 + 67 = 360$

$3X + 156 = 360$

$3X = 204$

$X = 68$

Find the measure of angle GFL in the polygon



$15b + 18b + 33b + 16b + 10b + 28b = 360$

$120b = 360$

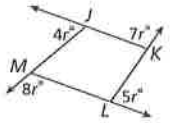
$b = 3$

$m\angle GFL = 180 - 15b = 180 - 45 = 135^\circ$

Find the measure of each exterior angle of a regular dodecagon.

$\frac{360}{12} = 30^\circ$

Find the value of r in polygon $JKLM$.



$4r + 7r + 5r + 8r = 360$

$24r = 360$

$r = 15$

Homework:

pg. 364 #4 - 30 Evens, 38, 39

In case you were wondering....

Polygons with 21 to 99 sides have a different system. Take the prefix for the tens digit (found on the left column), the ones digit (right column below), and then stick a "kai" between them to get `(tens)kai(ones)gon` .

10 - deca	1 - hena
20 - icosi	2 - di
30 - triaconta	3 - tri
40 - tetraconta	4 - tetra
50 - pentaconta	5 - penta
60 - hexaconta	6 - hexa
70 - heptaconta	7 - hepta
80 - octaconta	8 - octa
90 - nonaconta	9 - nona