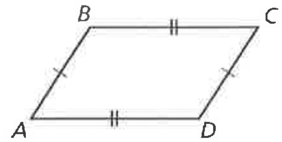
7.3 - Proving A Quadrilateral is a P-Gram

Theorem 7.7 Parallelogram Opposite Sides Converse

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$, then $ABCD$ is a parallelogram.

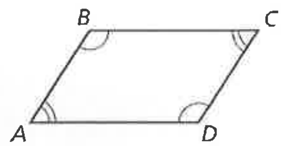


Theorem 7.8 Parallelogram Opposite Angles Converse

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.

Proof Ex. 39, p. 383



In quadrilateral $WXYZ$, $m\angle W = 42°$, $m\angle X = 138°$, and $m\angle Y = 42°$. Find $m\angle Z$.

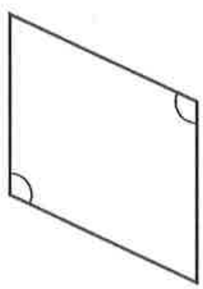
$360 - (42 + 138 + 42)$

$360 - 222 = 138°$

Is $WXYZ$ a parallelogram? Explain your reasoning.

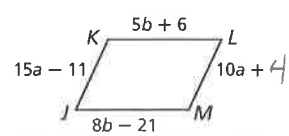
Yes, both pairs of opp. ∠'s are ≅

Determine if the quadrilateral must be a parallelogram. Justify your answer.



No, only one pair of opp. ∠'s ≅

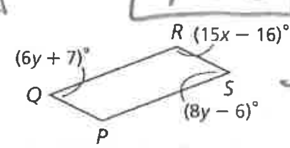
Find the values for a and b that make JKLM a parallelogram and state which condition you are using.



$15a - 11 = 10a + 4$
$-10a + 11 \quad -10a + 11$
$\frac{5a}{5} = \frac{15}{5}$ $\boxed{a = 3}$

$5b + 6 = 8b - 21$
$-5b + 21 \quad -5b + 21$
$27 = 3b$
$\boxed{9 = b}$

Find the values of x and y that make *PQRS* is a parallelogram and state which condition you are using.



$6y + 7 = 8y - 6$
$-6y + 6 \quad -6y + 6$
$13 = 2y$
$6.5 = y$
$m\angle Q = 6(6.5) + 7$
$= 39 + 7$
$= 46°$
$m\angle R = 180 - 46$
$= 134$

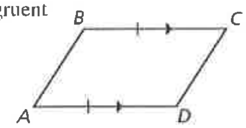
$15x - 16 = 134$
$15x = 150$
$x = 10$

Theorem 7.9 Opposite Sides Parallel and Congruent Theorem

If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.

If $\overline{BC} \parallel \overline{AD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.

Proof Ex. 40, p. 383

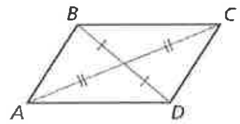


Theorem 7.10 Parallelogram Diagonals Converse

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

If $\overline{BD}$ and $\overline{AC}$ bisect each other, then $ABCD$ is a parallelogram.

Proof Ex. 41, p. 383

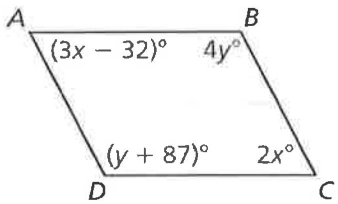


For what values of *x* and *y* is quadrilateral *ABCD* a parallelogram? Explain your reasoning.

$3x - 32 = 2x$

$-32 = -x$

$\boxed{32 = x}$

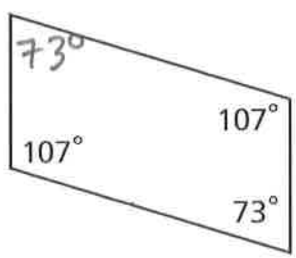


$4y = y + 87$

$3y = 87$

$\boxed{y = 29}$

Determine if the quadrilateral must be a parallelogram. Justify your answer.



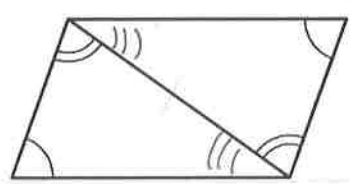
$360 - (107 + 107 + 73)$

$360 - 287$

$73°$

Yes - both opp. ∠'s are ≅

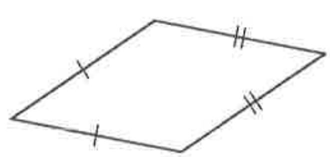
Determine if the quadrilateral must be a parallelogram. Justify your answer.



Other ∠'s are ≅ by third angus theorem

Both pair opp. ∠'s are ≅ → parallelogram

Determine if each quadrilateral must be a parallelogram. Justify your answer.



No

Show that quadrilateral *JKLM* is a parallelogram by using the definition of parallelogram. *J*(−1, −6), *K*(−4, −1), *L*(4, 5), *M*(7, 0).

→ opp sides are parallel

Slope of JK: $-3\left(\begin{matrix}-1,-6\\-4,-1\end{matrix}\right)+5$

$m=-\frac{5}{3}$

Slope of KL: $+8\left(\begin{matrix}-4 & -1\\4 & 5\end{matrix}\right)+6$

$m=\frac{6}{8}=\frac{3}{4}$

slope of JM: $+8\left(\begin{matrix}-1,-6\\7,0\end{matrix}\right)+6$

$m=\frac{6}{8}=\frac{3}{4}$

$\overline{KL} \parallel \overline{JM}$

Slope of LM: $+3\left(\begin{matrix}4,5\\7,0\end{matrix}\right)-5$

$m=\frac{-5}{3}$

$\overline{JK} \parallel \overline{LM}$

Show that quadrilateral ABCD is a parallelogram using the diagonals. A(2,3), B(6,2), C(5,0), and D(1,1)

*Show diagonals have same mdpt.

mdpt of AC: $\left(\frac{2+5}{2},\frac{3+0}{2}\right)$

$\boxed{\left(\frac{7}{2},\frac{3}{2}\right)}$

mdpt of BD: $\left(\frac{6+1}{2},\frac{2+1}{2}\right)$

$\boxed{\left(\frac{7}{2},\frac{3}{2}\right)}$

Use the definition of a parallelogram to show that the quadrilateral with vertices *K*(−3, 0), *L*(−5, 7), *M*(3, 5), and *N*(5, −2) is a parallelogram.

Slope of KL: $-2\left(\begin{matrix}-3,0\\-5,7\end{matrix}\right)+7$

$m=-\frac{7}{2}$

Slope of MN: $+2\left(\begin{matrix}3,5\\5,-2\end{matrix}\right)-7$

$m=-\frac{7}{2}$

Slope of NK: $-8\left(\begin{matrix}5,-2\\-3,0\end{matrix}\right)+2$

$m=\frac{-2}{8}=-\frac{1}{4}$

Slope of LM: $+8\left(\begin{matrix}-5,7\\3,5\end{matrix}\right)-2$

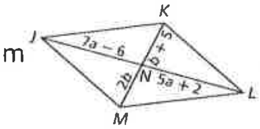
$m=\frac{-2}{8}=-\frac{1}{4}$

You have learned several ways to determine whether a quadrilateral is a parallelogram. You can use the given information about a figure to decide which condition is best to apply.

Conditions for Parallelograms
Both pairs of opposite sides are parallel. (definition)
One pair of opposite sides are parallel and congruent. (Theorem 6-3-1)
Both pairs of opposite sides are congruent. (Theorem 6-3-2)
Both pairs of opposite angles are congruent. (Theorem 6-3-3)
One angle is supplementary to both of its consecutive angles. (Theorem 6-3-4)
The diagonals bisect each other. (Theorem 6-3-5)

To show that a quadrilateral is a parallelogram, you only have to show that it satisfies one of these sets of conditions.

1. Show that *JKLM* is a parallelogram for $a = 4$ and $b = 5$.



$7a-6 = 5a+2$
$-5a+6 \quad -5a+6$

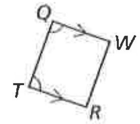
$\frac{2a}{2} = \frac{8}{2}$

$\boxed{a=4}$

$2b = b+5$
$-b \quad -b$

$\boxed{b=5}$

2. Determine if *QWRT* must be a parallelogram. Justify your answer.



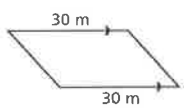
$\boxed{\text{No}}$

3. Show that the quadrilateral with vertices $E(-1, 5)$, $F(2, 4)$, $G(0, -3)$, and $H(-3, -2)$ is a parallelogram.

mdpt of EG: $\left(\frac{-1+0}{2}, \frac{-3+5}{2}\right) \rightarrow \left(\frac{-1}{2}, 1\right)$ mdpt of FH: $\left(\frac{2+(-3)}{2}, \frac{4+(-2)}{2}\right) \rightarrow \left(-\frac{1}{2}, 1\right)$

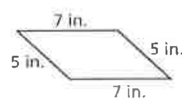
State the theorem you can use to show that the quadrilateral is a parallelogram.

3.



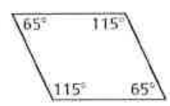
Yes, one opp. side ≅ and ∥

4.



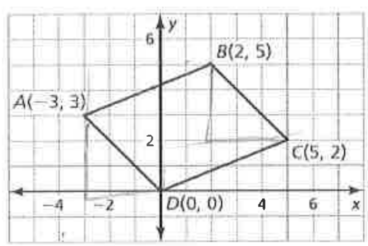
Yes, both opp sides ≅

5.



Yes, both opp ∠'s ≅

Show that quadrilateral *ABCD* is a parallelogram.



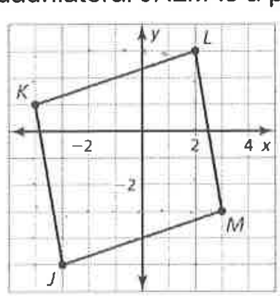
Slopes

$BC = \frac{-3}{3} = -1$ $AB = \frac{2}{5}$

$AD = \frac{-3}{3} = -1$ $DC = \frac{2}{5}$

Both pair opp sides parallel

Show that quadrilateral *JKLM* is a parallelogram.



Slopes

$KL = \frac{2}{6} = \frac{1}{3}$ $LM = \frac{-6}{1}$

$JM = \frac{2}{6} = \frac{1}{3}$ $KJ = \frac{-6}{1}$

Both pair opp sides parallel

Homework:

pg. 381 #3-8, 11-17, 19, 20, 23, 29, 35-36