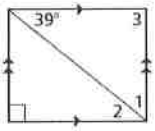
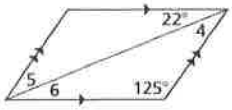
7.4 Special Parallelograms

Bellwork

Use the diagrams to determine the measure of each angle.





180 - 125 = 55°

1. $m\angle 1$ 90 - 39° = 51°
2. $m\angle 2$ 39°
3. $m\angle 3$ 90°
4. $m\angle 4$ 55 - 22 = 33°
5. $m\angle 5$ 33°
6. $m\angle 6$ 22°

Objectives

Prove and apply properties of rectangles, rhombuses, and squares.

Use properties of rectangles, rhombuses, and squares to solve problems.

Core Concept

Rhombuses, Rectangles, and Squares

A **rhombus** is a parallelogram with four congruent sides.

A **rectangle** is a parallelogram with four right angles.

A **square** is a parallelogram with four congruent sides and four right angles.

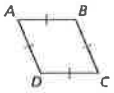
Corollaries

Corollary 7.2 Rhombus Corollary

A quadrilateral is a rhombus if and only if it has four congruent sides.

ABCD is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.

Proof Ex. 81, p. 396

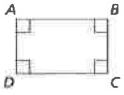


Corollary 7.3 Rectangle Corollary

A quadrilateral is a rectangle if and only if it has four right angles.

ABCD is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof Ex. 82, p. 396

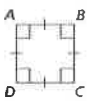


Corollary 7.4 Square Corollary

A quadrilateral is a square if and only if it is a rhombus and a rectangle.

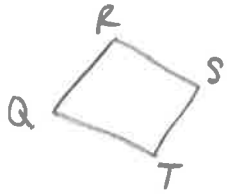
ABCD is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof Ex. 83, p. 396



For any rhombus *QRST*, decide whether the statement is *always* or *sometimes* true.

Draw a diagram and explain your reasoning.



a. $\angle Q \cong \angle S$ Always

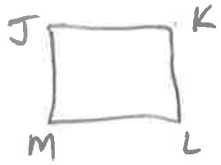
b. $\angle Q \cong \angle R$ Sometimes

Classify the special quadrilateral.

Explain your reasoning.

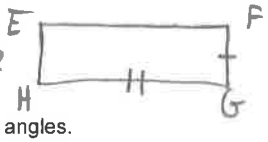
Rhombus - opp sides $\cong$ and consecutive sides $\cong$

1. For any square *JKLM*, is it *always* or *sometimes* true that $\overline{JK} \perp \overline{LM}$? Explain your reasoning.



Never - they are parallel

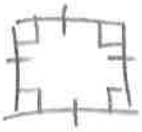
2. For any rectangle *EFGH*, is it *always* or *sometimes* true that $\overline{FG} \cong \overline{GH}$? Explain your reasoning.



Sometimes; when it is a square

3. A quadrilateral has four congruent sides and four congruent angles. Sketch the quadrilateral and classify it.

Square (and p-gram, rectangle, rhombus)

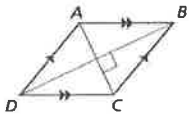


Theorem 7.11 Rhombus Diagonals Theorem

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

$\square ABCD$ is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.

Proof p. 390; Ex. 72, p. 395

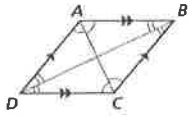


Theorem 7.12 Rhombus Opposite Angles Thoerem

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

$\square ABCD$ is a rhombus if and only if $\overline{AC}$ bisects $\angle BCD$ and $\angle BAD$, and $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$.

Proof Exs. 73 and 74, p. 395



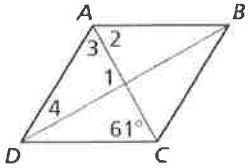
Find the measures of the numbered angles in rhombus *ABCD*.

$m\angle 1 = 90$

$m\angle 2 = 61°$

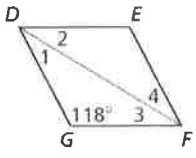
$m\angle 3 = 61°$

$m\angle 4 = 29°$



4. In Example 3, what is $m\angle ADC$ and $m\angle BCD$?

$m\angle ADC = 2(29) = 58°$



$m\angle BCD = 2(61) = 122°$

5. Find the measures of the numbered angles in rhombus *DEFG*.

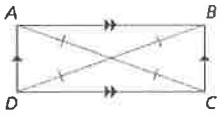
$180 - 118 = 62°$ $m\angle 1 = m\angle 2 = m\angle 3 = m\angle 4 = \frac{62}{2} = 31°$

Theorem 7.13 Rectangle Diagonals Theorem

A parallelogram is a rectangle if and only if its diagonals are congruent.

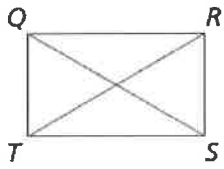
$\square ABCD$ is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.

Proof Exs. 87 and 88, p. 396



In rectangle $QRST$, $QS = 5x - 31$ and $RT = 2x + 11$.

Find the lengths of the diagonals of $QRST$.



$5x - 31 = 2x + 11$

$-2x + 31 \quad -2x + 31$

$\frac{3x}{3} = \frac{42}{3}$

$x = 14$

$QS = 5(14) - 31$

$= 39$

$RT = 2(14) + 11 = 39$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AC = \sqrt{(4+2)^2 + (0-6)^2}$

$AC = \sqrt{36 + 36} = \sqrt{72}$

$BD = \sqrt{(4-6)^2 + (-2-8)^2}$

$BD = \sqrt{(-10)^2 + (-10)^2}$

$BD = \sqrt{200}$

Diagonals not $\cong$

Not a Rectangle

Slope of AC $+6\left(\begin{matrix}-2,6\\4,0\end{matrix}\right)-6$

$m = \frac{-6}{6} = -1$

Slope of BD $-10\left(\begin{matrix}6,8\\-4,-2\end{matrix}\right)-10$

$m = \frac{-10}{-10} = 1$

$\overline{AC} \perp \overline{BD}$

Diagonals $\perp$ → Rhombus

Decide whether $\square ABCD$ with vertices $A(-2, 6)$, $B(6, 8)$, $C(4, 0)$, and $D(-4, -2)$ is a *rectangle*, a *rhombus*, or a *square*. Give all names that apply.

Rhombus

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$PR = \sqrt{(2+5)^2 + (-1-2)^2}$

$PR = \sqrt{49 + 9}$

$PR = \sqrt{58}$

8. Decide whether $\square PQRS$ with vertices $P(-5, 2)$, $Q(0, 4)$, $R(2, -1)$, and $S(-3, -3)$ is a *rectangle*, a *rhombus*, or a *square*. Give all names that apply.

Rectangle, Rhombus, Square

$QS = \sqrt{(-3-0)^2 + (-3-4)^2}$

$QS = \sqrt{9 + 49} = \sqrt{58}$

$\overline{PR} \cong \overline{QS}$ → Rectangle

Slope of PR $+7\left(\begin{matrix}-5,2\\2,-1\end{matrix}\right)-3$

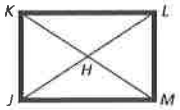
$m = -\frac{3}{7}$

Slope of QS $-3\left(\begin{matrix}0,4\\-3,-3\end{matrix}\right)-7$

$m = \frac{-7}{-3} = \frac{7}{3}$

$\overline{PR} \perp \overline{QS}$ → Rhombus

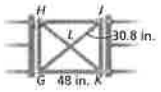
A woodworker constructs a rectangular picture frame so that $JK = 50$ cm and $JL = 86$ cm. Find HM.



$JL = 86$, so $KM = 86$

$HM = \frac{86}{2} = 43$

Carpentry The rectangular gate has diagonal braces. Find HJ.



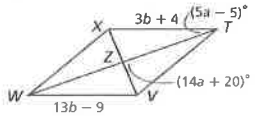
$HJ = 48''$

Find HK.

$JG = 2(30.8) = 61.6$

$HK = 61.6$

***TVWX* is a rhombus. Find *TV*.**



$3b + 4 = 13b - 9$

$13 = 10b$

$\frac{13}{10} = b$

$TV = 3\left(\frac{13}{10}\right) + 4 = \frac{39}{10} + 4 = \frac{39}{10} + \frac{40}{10} = \frac{79}{10}$

Find $m\angle VTZ$.

$14a + 20 = 90 \rightarrow 14a = 70 \rightarrow a = 5$

$m\angle VTZ = 5(5) - 5 = 20°$

Homework:

pg. 393 #3-8, 13-14, 16-25, 29-34, 37-56, 65 - 70