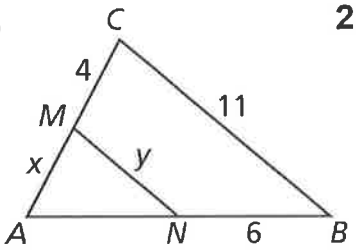
7.5 Kites and Trapezoids

Bellwork

$\overline{MN}$ is a midsegment of $\triangle ABC$. Find the values of x and y.

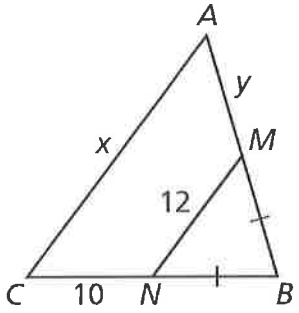
1.



$X = 4$

$Y = \frac{1}{2}(11) = 5.5$

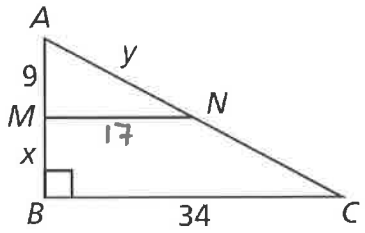
2.



$X = 2(12) = 24$

$Y = 10$

3.



$X = 9$ $MN = 17$

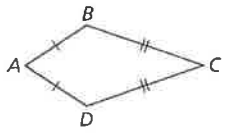
$9^2 + 17^2 = y^2$

$81 + 289 = y^2$

$370 = y^2$

$\sqrt{370} = y$

A **kite** is a quadrilateral with exactly two pairs of congruent consecutive sides.



Kite *ABCD*

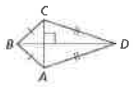
Theorems

Theorem 7.18 Kite Diagonals Theorem

If a quadrilateral is a kite, then its diagonals are perpendicular.

If quadrilateral *ABCD* is a kite, then $\overline{AC} \perp \overline{BD}$.

Proof p. 401

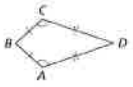


Theorem 7.19 Kite Opposite Angles Theorem

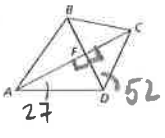
If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

If quadrilateral *ABCD* is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A \cong \angle C$ and $\angle B \ncong \angle D$.

Proof Ex. 47, p. 406



In kite *ABCD*, $m\angle DAB = 54°$, and $m\angle CDF = 52°$. Find $m\angle BCD$.



$m\angle DCF = 90 - 52 = 38$

$m\angle BCD = 2(38) = 76°$

Find $m\angle ABC$.

$m\angle ABF = 90 - 27 = 63°$

$m\angle ABC = 63 + 52 = 115°$

Find $m\angle FDA$.

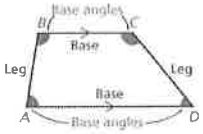
$m\angle FDA = 63°$

In kite *PQRS*, PQ = 5 and RS = 7. Find the perimeter of the kite.

$P = 5 + 5 + 7 + 7 = 24$

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides.

Bases - the parallel sides
Legs – the non parallel sides
Base angles - consecutive angles whose common side is a base.



isosceles trapezoid - legs of a trapezoid are congruent.

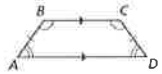
Theorems

Theorem 7.14 Isosceles Trapezoid Base Angles Theorem

If a trapezoid is isosceles, then each pair of base angles is congruent.

If trapezoid $ABCD$ is isosceles, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.

Proof Ex. 39, p. 405

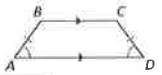


Theorem 7.15 Isosceles Trapezoid Base Angles Converse

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid $ABCD$ is isosceles.

Proof Ex. 40, p. 405

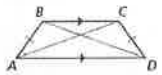


Theorem 7.16 Isosceles Trapezoid Diagonals Theorem

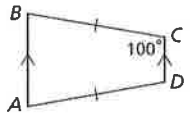
A trapezoid is isosceles if and only if its diagonals are congruent.

Trapezoid $ABCD$ is isosceles if and only if $\overline{AC} \cong \overline{BD}$.

Proof Ex. 51, p. 406



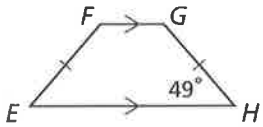
Find m∠A.



$m\angle A = 180 - 100$

$= 80°$

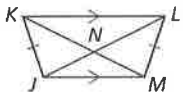
Find m∠F.



$m\angle F = 180 - 49$

$= 131°$

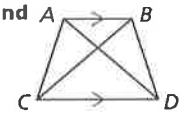
JN = 10.6, and NL = 14.8. Find KM.



$KM = 10.6 + 14.8$

$KM = 25.4$

AD = 12x – 11, and BC = 9x – 2. Find the value of x so that ABCD is isosceles.



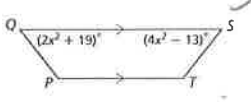
$12x - 11 = 9x - 2$

$3x - 11 = -2$

$3x = 9$

$x = 3$

Find the value of x so that PQST is isosceles.



$2x^2 + 19 = 4x^2 - 13$

$32 = 2x^2$

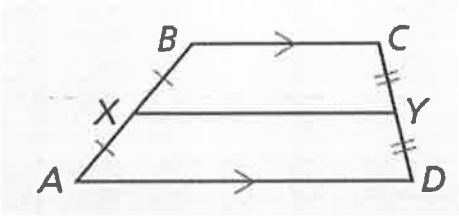
$16 = x^2$

$\pm 4 = x$

The **midsegment of a trapezoid** is the segment whose endpoints are the midpoints of the legs

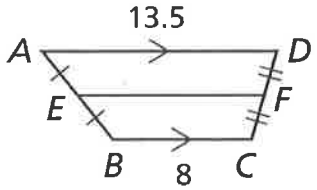
Trapezoid Midsegment Theorem

- Parallel to each base
- Length is the average of the bases



$$XY = \frac{1}{2}(BC + AD)$$

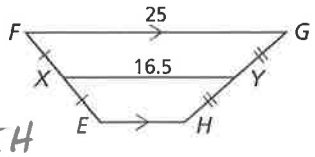
Find *EF*.



$EF = \frac{1}{2}(13.5 + 8)$

$EF = 10.75$

Find *EH*.

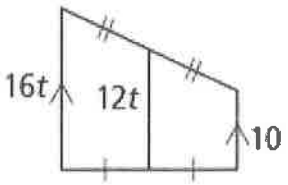


$XY = \frac{1}{2}(FG + EH)$

$16.5 = \frac{1}{2}(25 + EH)$ → $33 = 25 + EH$

$8 = EH$

Solve for t.



$12t = \frac{1}{2}(16t + 10)$

$12t = 8t + 5$

$4t = 5$

$t = \frac{5}{4}$

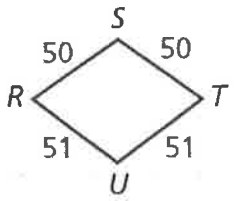
Quadrilateral *DEFG* has at least one pair of opposite sides congruent. What types of quadrilaterals meet this condition?

parallelograms (inc. rect., rhombus, squares)

isosceles trapezoids

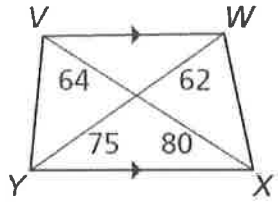
Give the most specific name for the quadrilateral. Explain your reasoning.

8.



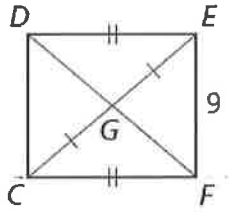
kite

9.



trapezoid

10.



quadrilateral

Homework:

pg. 403 #7-12, 15-18, 21-28, 31-34