Chapter 8 - Similarity

8.1 Similar Polygons

Bellwork - Solve the proportion.

1. $\frac{12}{x}=\frac{3}{5}$

$3x=60$

$x=20$

2. $\frac{x}{9}=\frac{1}{x}$

$x^2=9$

$x=\pm 3$

3. $\frac{4-x}{12}=\frac{3}{-7}$

$-7(4-x)=36$

$-28+7x=36$

$+28 \quad +28$

$7x=64$

$x=9.14$

4. $\frac{x+3}{2}=\frac{3}{5}$

$5(x+3)=6$

$5x+15=6$

$\frac{5x}{5}=\frac{-9}{5}$

$x=\frac{-9}{5}$

$\frac{x-3}{4}=\frac{2}{x-3}$

$\sqrt{(x-3)^2}=\sqrt{8}$

$x-3=\pm 2\sqrt{2}$

$x=3\pm 2\sqrt{2}$

5. $\frac{1}{2x+1}=\frac{x-3}{9}$

$9=(2x+1)(x-3)$

$9=2x^2+1x-6x-3$

$9=2x^2-5x-3$

$0=2x^2-5x-12$

$0=(x-4)(2x+3)$

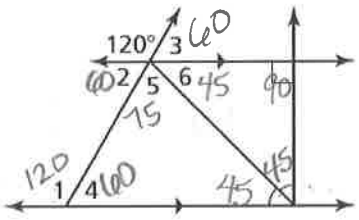
$x-4=0 \qquad 2x+3=0$

$2x=-3$

$x=4$

$x=\frac{-3}{2}$

Use the diagram to find the measure of the angle.



1. $\angle 1=120^\circ$

2. $\angle 2=60$

3. $\angle 3=60$

4. $\angle 4=60^\circ$

5. $\angle 5=75^\circ$

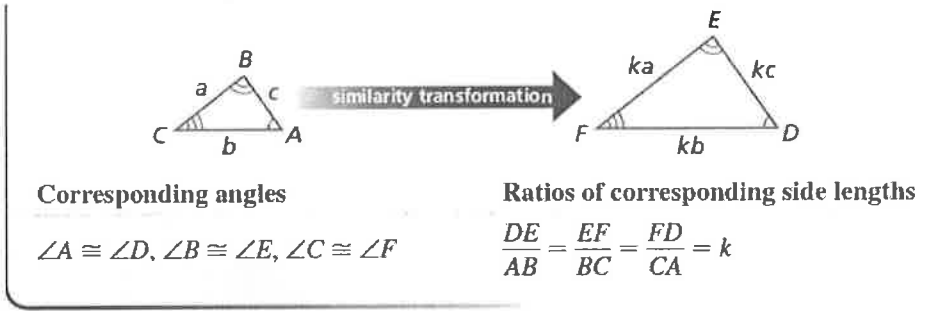
6. $\angle 6=45^\circ$

Corresponding Parts of Similar Polygons

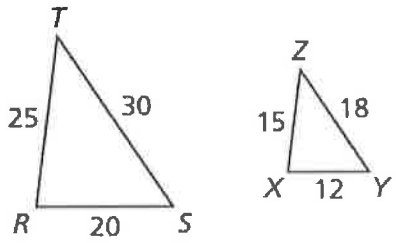
Triangle ABC is similar to triangle DEF

$\Delta ABC \sim \Delta DEF$

1. Corresponding angles are congruent
2. Sides are enlarged or reduced by a scale factor, k (corresponding sides are proportional)



In the diagram, $\triangle RST \sim \triangle XYZ$.



a. Find the scale factor from $\triangle RST$ to $\triangle XYZ$.

$\frac{XY}{RS} = \frac{12}{20} = \boxed{\frac{3}{5}}$

b. List all pairs of congruent angles.

$\angle A \cong \angle J \quad \angle B \cong \angle K \quad \angle C \cong \angle L$

c. Write the ratios of the corresponding side lengths in a *statement of proportionality.*

$\frac{XZ}{RT} = \frac{ZY}{TS} = \frac{YX}{SR}$

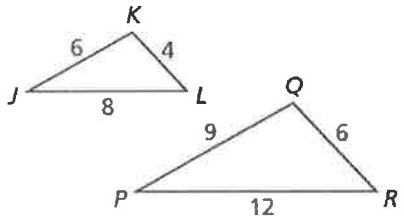
In the diagram, $\triangle JKL \sim \triangle PQR$.

1. Find the scale factor from $\triangle JKL$ to $\triangle PQR$. $\frac{PR}{JL} = \frac{12}{8} = \boxed{\frac{3}{2}}$

2. List all pairs of congruent angles

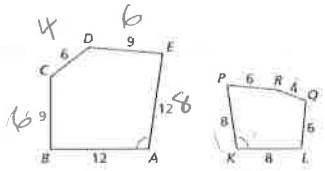
$\angle J \cong \angle P$; $\angle L \cong \angle R$; $\angle K \cong \angle Q$

3. Write the ratios of the corresponding side lengths in a statement of proportionality.



$\frac{PQ}{JK} \cong \frac{QR}{KL} \cong \frac{RP}{LJ}$

Decide whether *ABCDE* and *KLQRP* are similar. Explain your reasoning.



$\frac{6}{9} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{8}{12}$

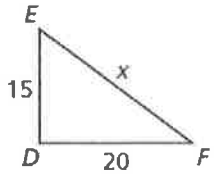
$\frac{2}{3} = \frac{2}{3} = \frac{2}{3} = \frac{2}{3} = \frac{2}{3}$ all ratio's equal

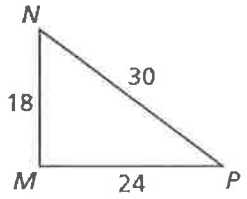
Corresponding Lengths in Similar Polygons

If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the scale factor of the similar polygons.

In the diagram, $\triangle DEF \sim \triangle MNP$.

Find the value of x.





$\frac{ED}{Nm} = \frac{EF}{NP}$

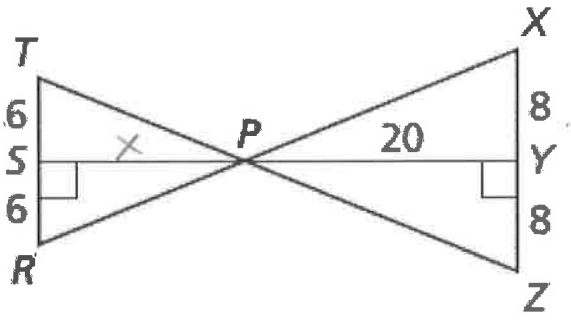
$\frac{15}{18} = \frac{x}{30}$

$450 = 18x$

$25 = x$

In the diagram, $\triangle TPR \sim \triangle XPZ$.

Find the length of the altitude $\overline{PS}$.



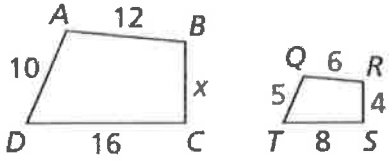
$\frac{6}{8} = \frac{x}{20}$

$120 = 8x$

$15 = x$

$\overline{PS} = 15$

2. Find the value of x.



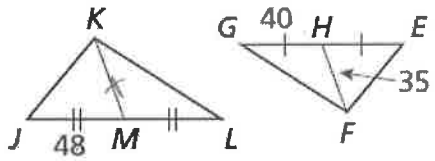
$ABCD \sim QRST$

$\frac{x}{4} = \frac{16}{8}$

$64 = 8x$

$8 = x$

3. Find KM.



$\triangle JKL \sim \triangle EFG$

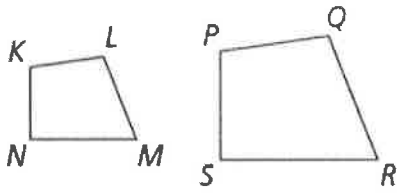
$\frac{40}{48} = \frac{35}{x}$

$1680 = 40x$

$42 = x$

Theorem 8.1 Perimeters of Similar Polygons

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.



If $KLMN \sim PQRS$, then $\frac{PQ + QR + RS + SP}{KL + LM + MN + NK} = \frac{PQ}{KL} = \frac{QR}{LM} = \frac{RS}{MN} = \frac{SP}{NK}$.

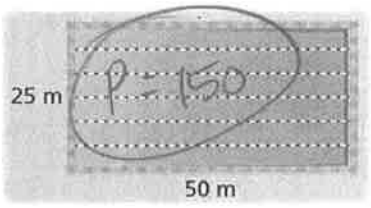
Proof Ex. 52, p. 426; *BigIdeasMath.com*

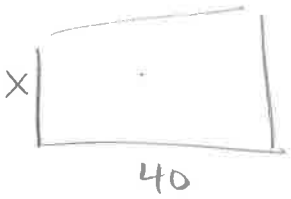
A town plans to build a new swimming pool. An Olympic pool is rectangular with a length of 50 meters and a width of 25 meters. The new pool will be similar in shape to an Olympic pool but will have a length of 40 meters. Find the perimeters of an Olympic pool and the new pool.

$\frac{40}{50} = \frac{P}{150}$

$5P = 600$

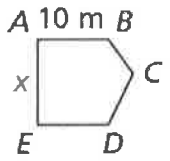
$P = 120$



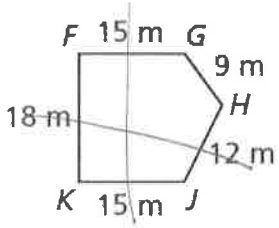


The two gazebos shown are similar pentagons. Find the perimeter of Gazebo A.

Gazebo A



Gazebo B

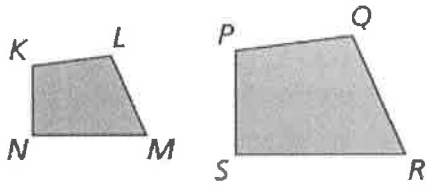


$\frac{10}{15} = \frac{P}{69} = \frac{2}{3}$

$138 = 3P$

$46 = P$

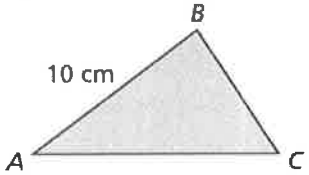
If two polygons are similar, then the ratio of their areas is equal to the squares of the ratios of their corresponding side lengths.



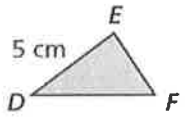
If $KLMN \sim PQRS$, then $\frac{\text{Area of } PQRS}{\text{Area of } KLMN} = \left(\frac{PQ}{KL}\right)^2 = \left(\frac{QR}{LM}\right)^2 = \left(\frac{RS}{MN}\right)^2 = \left(\frac{SP}{NK}\right)^2$.

Proof Ex. 53, p. 426; *BigIdeasMath.com*

In the diagram, $\triangle ABC \sim \triangle DEF$. Find the area of $\triangle DEF$.



Area of $\triangle ABC = 36$ cm^2



$\left(\frac{5}{10}\right)^2 = \left(\frac{A}{36}\right)^2$

$\frac{5^2}{10^2} = \frac{A^2}{36^2}$

$\frac{25}{100} = \frac{a^2}{1296}$

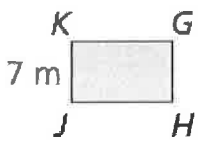
$\frac{1}{4}$

$1296 = 4a^2$

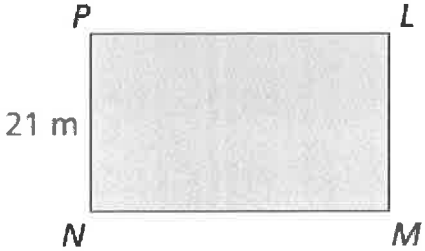
$324 = a^2$

$18 = a$

In the diagram, $GHJK \sim LMNP$. Find the area of $LMNP$.



Area of $GHJK = 84$ m^2



$\left(\frac{7}{21}\right)^2 = \left(\frac{84}{A}\right)^2$

$\frac{7^2}{21^2} = \frac{84^2}{a^2}$

$\frac{49}{441} = \frac{84^2}{A^2}$

$a^2 = 63504$

$a = 252$ m^2

Homework:

pg. 423 #4-22 Evens, 37, 40-41