# 8.4 Proportionality Theorems

**Bellwork - Solve the proportion.**

**1.** $\frac{x-1}{3}=\frac{2x}{5}$

$6x = 5x - 5$
$-5x \quad -5x$
$x = -5$

**2.** $\frac{2x+3}{3x}=\frac{4}{9}$

$12x = 9(2x+3)$
$12x = 18x + 27$
$-6x = 27$
$x = -4.5$

**3.** $\frac{2x}{x+3}=\frac{3x}{x-3}$

$2x(x-3) = 3x(x+3)$
$2x^2 - 6x = 3x^2 + 9x$
$0 = x^2 + 15x$
$0 = x(x+15)$
$\boxed{x = 0, -15}$

**4.** $\frac{x}{2x+1}=\frac{5}{4-x}$

$x(4-x) = 5(2x+1)$
$4x - x^2 = 10x + 5$
$0 = x^2 + 6x + 5$
$0 = (x+5)(x+1)$
$x = -5, -1$

**5.** $\frac{2}{1-x}=\frac{x-8}{x+1}$

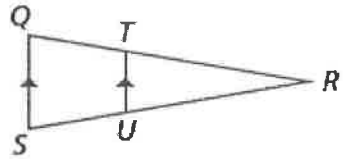
$2(x+1) = (1-x)(x-8)$
$2x + 2 = x - x^2 - 8 + 8x$
$0 = -x^2 + 7x - 10$
$0 = x^2 - 7x + 10$
$0 = (x-2)(x-5)$
$\boxed{x = 2, 5}$

**6.** $\frac{x}{2x-6}=\frac{2}{x-4}$

$x(x-4) = 2(2x-6)$
$x^2 - 4x = 4x - 12$
$x^2 - 8x + 12 = 0$
$(x-2)(x-6) = 0$
$\boxed{x = 2, 6}$

**Theorem 8.6 Triangle Proportionality Theorem**

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

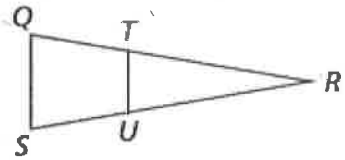



If $\overline{TU} \parallel \overline{QS}$, then $\frac{RT}{TQ} = \frac{RU}{US}$.

*Proof* Ex. 27, p. 451

**Theorem 8.7 Converse of the Triangle Proportionality Theorem**

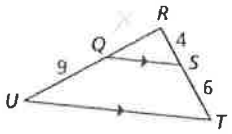
If a line divides two sides of a triangle proportionally, then it is parallel to the third side.




If $\frac{RT}{TQ} = \frac{RU}{US}$, then $TU \parallel QS$.

*Proof* Ex. 28, p. 451

In the diagram, $\overline{QS} \parallel \overline{UT}$, $RS = 4$, $ST = 6$, and $QU = 9$. What is the length of $\overline{RQ}$?



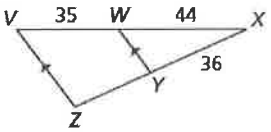

$$\frac{RS}{ST} = \frac{RQ}{QU}$$

$$\frac{4}{6} = \frac{RQ}{9}$$

$$36 = 6(RQ)$$

$$\boxed{RQ = 6}$$

Find the length of $\overline{YZ}$.



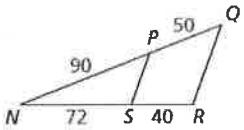

$$\frac{XW}{WV} = \frac{XY}{YZ}$$

$$\frac{44}{35} = \frac{36}{YZ}$$

$$1260 = 44(YZ)$$

$$28.64 \approx YZ$$

Determine whether $\overline{PS} \parallel \overline{QR}$.




$$\frac{50}{90} \stackrel{?}{=} \frac{40}{72}$$

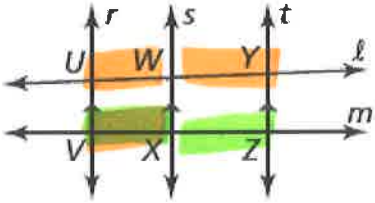
$$(40)90 = 72(50)$$

$$3600 = 3600$$

yes

## Theorem 8.8 Three Parallel Lines Theorem

**If three parallel lines intersect two transversals, then they divide the transversals proportionally.**

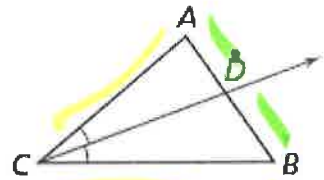



$$\frac{UW}{WY} = \frac{VX}{XZ}$$

*Proof* Ex. 32, p. 451

## Theorem 8.9 Triangle Angle Bisector Theorem

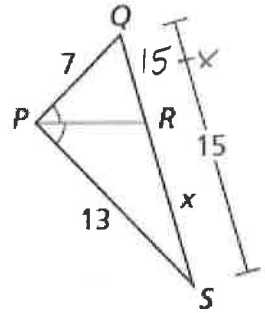
If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.




$$\frac{AD}{DB} = \frac{CA}{CB}$$

*Proof* Ex. 35, p. 452

In the diagram, $\angle QPR \cong \angle RPS$. Use the given side lengths to find the length of $\overline{RS}$.




$$\frac{QR}{RS} = \frac{PQ}{PS}$$

$$\frac{15-x}{x} = \frac{7}{13}$$

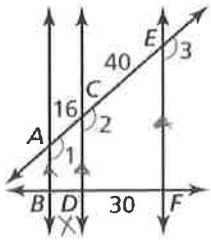
$$7x = 195 - 13x$$

$$20x = 195$$

$$x = 9.75$$

**Find the length of the given line segment.**

**3.** $\overline{BD}$

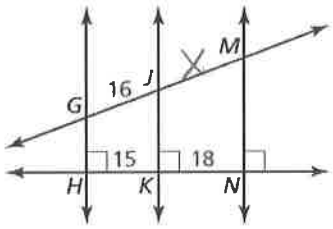



$$\frac{BD}{30} = \frac{16}{40}$$

$$480 = 40(BD)$$

$$12 = BD$$

**4.** $\overline{JM}$



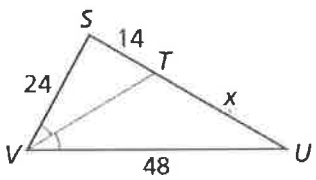

$$\frac{16}{JM} = \frac{15}{18}$$

$$288 = 15(JM)$$

$$19.2 = JM$$

**Find the value of the variable.**

**5.**

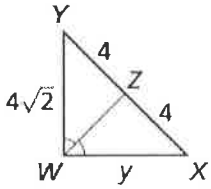



$$\frac{14}{x} = \frac{24}{48}$$

$$672 = 24x$$

$$28 = x$$

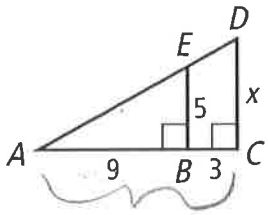
**6.**




$$\frac{4}{4} = \frac{4\sqrt{2}}{y}$$

$$y = 4\sqrt{2}$$

**Explain why $\Delta ABE \sim \Delta ACD$, and then find $CD$.**




$\angle B \cong \angle C$

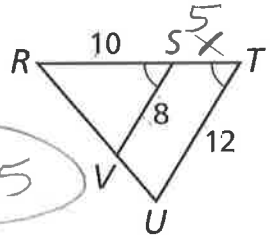
$\angle A \cong \angle A$ Reflex.

AA Sim. Thrm.

$\frac{9}{7} = \frac{5}{x}$

$x = 6\frac{2}{3}$

Explain why $\Delta RSV \sim \Delta RTU$ and then find $RT$.




$\frac{8}{12} = \frac{10}{10+x}$

$120 = 80 + 8x$

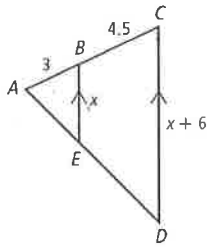
$-80 \quad -80$

$40 = 8x$

$x = 5$

$RT = 15$

Explain why the triangles are similar, then find $BE$ and $CD$.




$\frac{3}{7.5} = \frac{x}{x+6}$

$7.5x = 3x + 18$

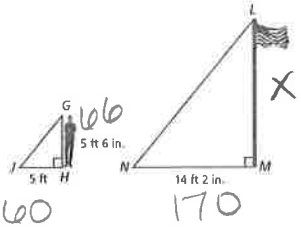
$4.5x = 18$

$x = 4$

$BE = 4$

$CD = 10$

**A student who is 5 ft 6 in. tall measured shadows to find the height $LM$ of a flagpole. What is $LM$?**



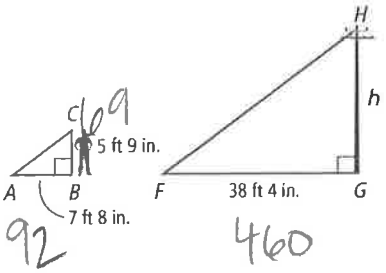

$\frac{66}{x} = \frac{60}{170}$

$11{,}220 = 60x$

$187 = x$

$LM = 187$ inches

**Tyler wants to find the height of a telephone pole. He measured the pole's shadow and his own shadow and then made a diagram. What is the height $h$ of the pole?**

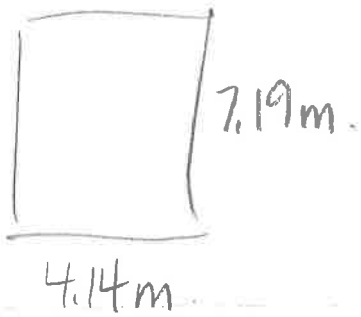



$\frac{69}{h} = \frac{92}{460}$

$92h = 31740$

$h = 345$ in.

**Lady Liberty holds a tablet in her left hand. The tablet is 7.19 m long and 4.14 m wide. If you made a scale drawing using the scale 1 cm:0.75 m, what would be the dimensions to the nearest tenth?**




$$\frac{1\text{cm}}{.75\text{m}} = \frac{x}{7.19\text{m}}$$

$$7.19 = .75x$$

$$9.58\overline{6}\text{cm} = x$$

$$\frac{1\text{cm}}{.75\text{m}} = \frac{w}{4.14\text{m}}$$

$$4.14 = .75w$$

$$5.52 = w$$

$$\boxed{9.58\overline{6}\text{cm by } 5.52\text{cm}}$$

Homework:

pg. 450 #4-8 Evens, 13-17, 20-22,25,26,30