9.1 - Pythagorean Theorem

Bellwork: Solve the equation.

1. $4^2 + 3^2 = x^2$

$16 + 9 = x^2$

$\sqrt{25} = \sqrt{x^2}$

$\pm 5 = x$

2. $13^2 + x^2 = 25^2$

$169 + x^2 = 625$

$\sqrt{x^2} = \sqrt{456}$

$\sqrt{4}\ \sqrt{114}$

$2\ \sqrt{57}$

$x = \pm 2\sqrt{114}$

3. $\left(\frac{5}{2}\right)^2 + x^2 = \left(\frac{1}{3}\right)^2$

$\frac{9}{9} \cdot \frac{25}{4} + x^2 = \frac{1}{9} \cdot \frac{4}{4}$

$-\frac{225}{36} \qquad -\frac{225}{36}$

$x^2 = -\frac{221}{36}$

No Real #'s

4. $\left(9\sqrt{3}\right)^2 - x^2 = 2^2$ $3\ 19$

$81 \cdot 3 - x^2 = 4$

$243 - x^2 = 4$

$-x^2 = -239$

$x^2 = 239$

$x = \pm\sqrt{239}$

5. $\left(\sqrt{5}\right)^2 + x^2 = 12^2$

$5 + x^2 = 144$

$x^2 = 139$

$x = \pm\sqrt{139}$

6. $\left(5\sqrt{10}\right)^2 - \left(\sqrt{2}\right)^2 = x^2$

$25 \cdot 10 - 2 = x^2$

$\sqrt{248} = \sqrt{x^2}$

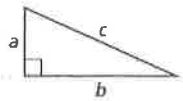
$\sqrt{4}\ \sqrt{62} = x$

$x = \pm 2\sqrt{62}$

Theorem 9.1 Pythagorean Theorem

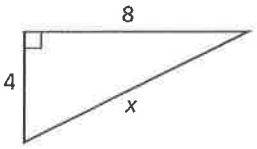
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

Proof Explorations 1 and 2, p. 463; Ex. 39, p. 484



$c^2 = a^2 + b^2$

Solve for x.



$4^2 + 8^2 = x^2$

$16 + 64 = x^2$

$\sqrt{80} = \sqrt{x^2}$

$\sqrt{5}\ \sqrt{16} = x$

$4\sqrt{5} = x$

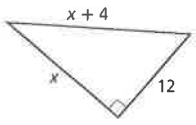
$x^2 + 12^2 = (x+4)^2$

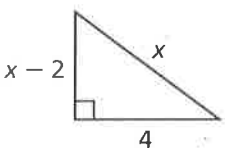
$x^2 + 144 = x^2 + 8x + 16$

$-x^2 \quad -16 \quad -x^2 \quad -16$

$128 = 8x$

$16 = x$





$(x-2)^2 + 4^2 = x^2$

$x^2 - 4x + 4 + 16 = x^2$

$-x^2 \qquad\qquad -x^2$

$-4x + 20 = 0$

$-4x = -20$

$x = 5$

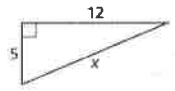
Common Pythagorean Triples and Some of Their Multiples

3, 4, 5	5, 12, 13	8, 15, 17	7, 24, 25
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
$3x, 4x, 5x$	$5x, 12x, 13x$	$8x, 15x, 17x$	$7x, 24x, 25x$

The most common Pythagorean triples are in bold. The other triples are the result of multiplying each integer in a bold-faced triple by the same factor.

Find the value of x. Then tell whether the side lengths form a Pythagorean triple.

5, 12, 13

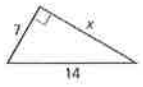


Find the value of x. Then tell whether the side lengths form a Pythagorean triple.

$7^2 + x^2 = 14^2$

$49 + x^2 = 196$

$\sqrt{x^2} = \sqrt{147}$

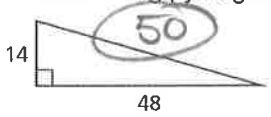


$x = \sqrt{49}\sqrt{3}$

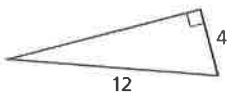
$x = 7\sqrt{3}$

Not a triplet

Determine if the third side length could be found using pythagorean triplets.



(7-24-25)



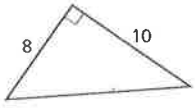
$4^2 + x^2 = 12^2$

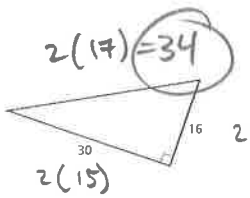
$16 + x^2 = 144$

$\sqrt{x^2} = \sqrt{128}$

$x = 8\sqrt{2}$

NO



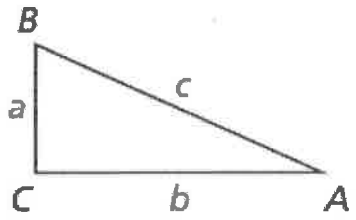


2(8)

(8-15-17)

Converse of the Pythagorean Theorem:

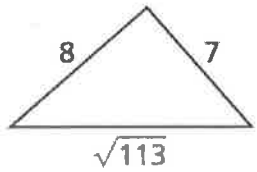
If $c^2 = a^2 + b^2$, then ABC is a right triangle.



** use the longest side as "c"

Tell whether each triangle is a right triangle.

a.



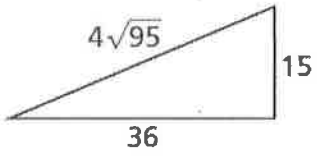
$7^2 + 8^2 \stackrel{?}{=} (\sqrt{113})^2$

$49 + 64 \stackrel{?}{=} 113$

$113 = 113$

Right △

b.



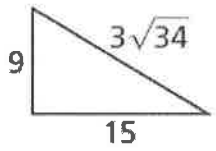
$15^2 + 36^2 \stackrel{?}{=} (4\sqrt{95})^2$

$225 + 1296 \stackrel{?}{=} 1520$

$1521 \neq 1520$

Not a Right △

Tell whether the triangle is a right triangle.

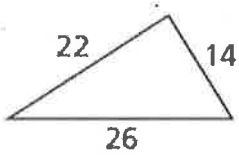


$9^2 + 15^2 \stackrel{?}{=} (3\sqrt{34})^2$

$81 + 225 \stackrel{?}{=} 306$

$306 = 306$

Right △



$22^2 + 14^2 \stackrel{?}{=} 26^2$

$484 + 196 \stackrel{?}{=} 676$

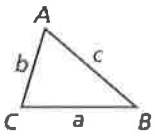
$680 \neq 676$

Not a Right △

Theorem 9.3 Pythagorean Inequalities Theorem

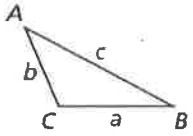
For any $\triangle ABC$, where c is the length of the longest side, the following statements are true.

If $c^2 < a^2 + b^2$, then $\triangle ABC$ is acute.



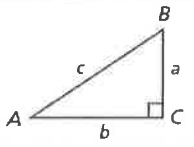
$c^2 < a^2 + b^2$

If $c^2 > a^2 + b^2$, then $\triangle ABC$ is obtuse.

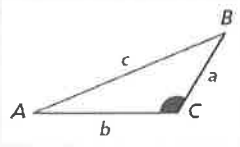


$c^2 > a^2 + b^2$

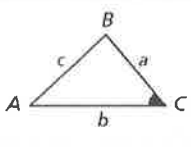
If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle by the Converse of the Pythagorean Theorem. So $m\angle C = 90°$.



If $c^2 > a^2 + b^2$, then c has increased. By the Converse of the Hinge Theorem, $m\angle C$ has also increased. So $m\angle C > 90°$.



If $c^2 < a^2 + b^2$, then c has decreased. By the Converse of the Hinge Theorem, $m\angle C$ has also decreased. So $m\angle C < 90°$.



Verify the segments form a triangle. If so, is the triangle, acute, right or obtuse?

4.3 feet, 5.2 feet, and 6.1 feet

$4.3 + 5.2 > 6.1$ ✓ Forms a △

$a^2 + b^2 \quad c^2$

$(4.3)^2 + (5.2)^2 \;?\; (6.1)^2$

$18.49 + 27.04 \quad 37.21$

$45.53 > 37.21$

Acute

3, 4, and 6

$3+4>6$ ✓ Triangle

$3^2 + 4^2 \;?\; 6^2$

$9 + 16 \quad 36$

$25 < 36$

Obtuse

2.1, 2.8, and 3.5

$2.1 + 2.8 > 3.5$ ✓ Triangle

$2.1^2 + 2.8^2 \;?\; 3.5^2$

$4.41 + 7.84 \quad 12.25$

$12.25 = 12.25$

Right

Homework:

pg. 468 #5-10, 17-19, 24,25,27,28
31,32,37