9.7A Law of Sines

Solve the proportion. Round your answer to the nearest tenth.

1. $\dfrac{a}{\sin 28°} = \dfrac{21}{\sin 65°}$

2. $\dfrac{110}{\sin C} = \dfrac{85}{\sin 36°}$

3. $\dfrac{b}{\sin 9°} = \dfrac{63}{\sin 105°}$

4. $\dfrac{54}{\sin B} = \dfrac{61}{\sin 73°}$

(1) $a \sin 65 = 21 \sin 28$

$a = \dfrac{21 \sin 28}{\sin 65} = \boxed{10.9}$

(2) $85 \sin C = 110 \sin 36$

$\sin C = \dfrac{110 \sin 36}{85}$

$m\angle C = \sin^{-1}\left(\dfrac{110 \sin 36}{85}\right) = \boxed{49.5°}$

(3) $b \sin 105 = 63 \sin 9$

$b = \dfrac{63 \sin 9}{\sin 105} = \boxed{10.2}$

(4) $61 \sin B = 54 \sin 73$

$\sin B = \dfrac{54 \sin 73}{61}$

$m\angle B = \sin^{-1}\left(\dfrac{54 \sin 73}{61}\right) = \boxed{57.8°}$

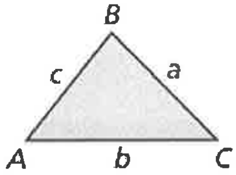
Use a calculator to find each trigonometric ratio. Round your answer to four decimal places.

a. tan 150° **b.** sin 120° **c.** cos 95°

$-.5774$ $.8660$ $-.0872$

Area of a Triangle

The area of any triangle is given by one-half the product of the lengths of two sides times the sine of their included angle. For $\triangle ABC$ shown, there are three ways to calculate the area.



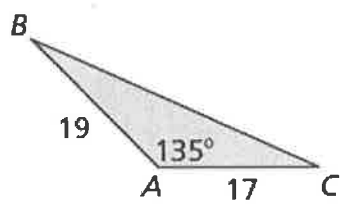
Area $= \frac{1}{2}bc \sin A$ Area $= \frac{1}{2}ac \sin B$ Area $= \frac{1}{2}ab \sin C$

Proof of Area Formula:

A = (1/2) base x height

in the triangle above, b would be the base and the height would be equivalent to c(sinA)

Find the area of each triangle. Round your answer to the nearest tenth.



$A = \frac{1}{2} bc \sin A$

$= \frac{1}{2}(17)(19)\sin 135$

$= \boxed{114.2 \text{ units}^2}$

Find the area of $\triangle ABC$ with the given side lengths and included angle. Round your answer to the nearest tenth.

4. $B = 60°$, $a = 19$, $c = 14$

$A = \frac{1}{2} ac \sin B$

$A = \frac{1}{2}(19)(14)\sin 60$

$\boxed{A = 115.2 \text{ units}^2}$

5. $C = 29°$, $a = 38$, $b = 31$

$A = \frac{1}{2} ab \sin C$

$A = \frac{1}{2}(38)(31)\sin 29$

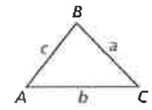
$\boxed{A = 285.6 \text{ units}^2}$

Theorem 9.9 Law of Sines

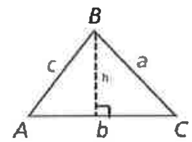
The Law of Sines can be written in either of the following forms for $\triangle ABC$ with sides of length a, b, and c.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \qquad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Proof Ex. 51, p. 516



Proof of Law of Sines



Sin A = h/c Sin C = h/a

Therefore, h = cSinA and h = aSinC

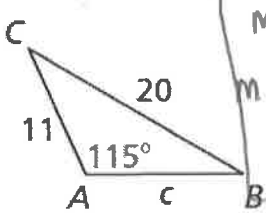
Then, cSinA = aSinC

and more simplification reveals

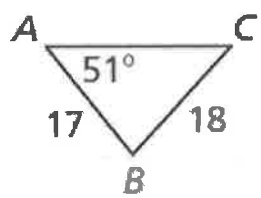
that SinA/a = SinC/c.

This can be done by drawing any of the three altitudes in the triangle.

Solve the triangle. Round decimal answers to the nearest tenth.



$m\angle B = 29.9°$
$m\angle C = 35.1°$
$c = 12.7$



$m\angle C = 47.2°$
$m\angle B = 81.8°$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 115}{20} = \frac{\sin B}{11}$$

$$\sin B = \frac{11 \sin 115}{20}$$

$$m\angle B = \sin^{-1}\left(\frac{11 \sin 115}{20}\right) = 29.9°$$

$$\frac{\sin 115}{20} = \frac{\sin 35.1}{c}$$

$$c = \frac{20 \sin 35.1}{\sin 115}$$

$$c = 12.7$$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 51}{18} = \frac{\sin C}{17}$$

$$\sin C = \frac{17 \sin 51}{18}$$

$$m\angle C = \sin^{-1}(\text{ans})$$

$$m\angle C = 47.2°$$

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

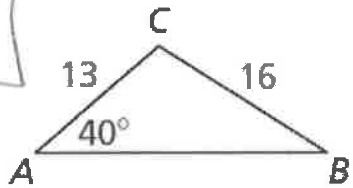
$$\frac{\sin 81.8}{b} = \frac{\sin 51}{18}$$

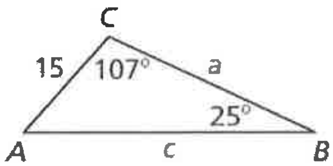
$$b = \frac{18 \sin 81.8}{\sin 51}$$

$$\boxed{b = 22.9}$$

Solve the triangle. Round decimal answers to the nearest tenth.

$\boxed{m\angle B = 31.5^\circ \quad m\angle C = 108.5^\circ}$





$m\angle A = 48^\circ$

$$\frac{\sin A}{a} = \frac{\sin B}{b} \qquad \frac{c}{\sin C} = \frac{a}{\sin A} \qquad \frac{c}{\sin C} = \frac{b}{\sin B} \qquad \frac{a}{\sin A} = \frac{b}{\sin B}$$

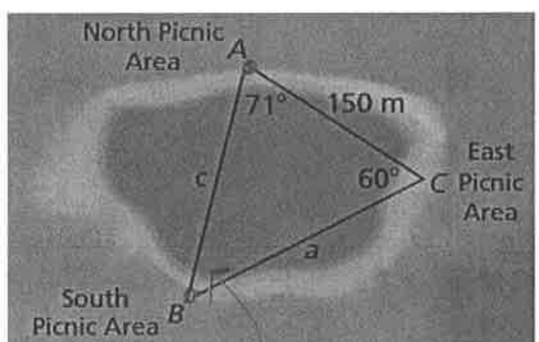
$$\frac{\sin 40}{16} = \frac{\sin B}{13} \qquad \frac{c}{\sin 108.5} = \frac{16}{\sin 40} \qquad \frac{c}{\sin 107} = \frac{15}{\sin 25} \qquad \frac{a}{\sin 48} = \frac{15}{\sin 25}$$

$$\sin B = \frac{13 \sin 40}{16} \qquad c = \frac{16 \sin 108.5}{\sin 40} \qquad c = \frac{15 \sin 107}{\sin 25} \qquad a = \frac{15 \sin 48}{\sin 25}$$

$$m\angle B = \sin^{-1}(\text{ans}) \qquad \boxed{c = 23.6} \qquad \boxed{c = 33.9} \qquad \boxed{a = 26.4}$$

$$m\angle B = 31.5^\circ$$

A surveyor makes the measurements shown to determine the length of a bridge to be built across a small lake from the North Picnic Area to the South Picnic Area. Find the length of the bridge.



49°

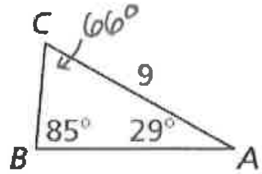
$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

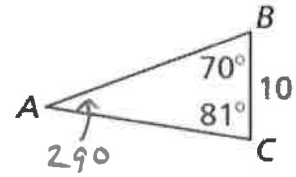
$$\frac{150}{\sin 49} = \frac{c}{\sin 60}$$

$$c = \frac{150 \sin 60}{\sin 49} = \boxed{172.1 \text{ m}}$$

Find the area of the triangles.

~~Solve the triangle:~~ Round decimal answers to the nearest tenth.





$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{c}{\sin 66} = \frac{9}{\sin 85}$$

$$c = \frac{9 \sin 66}{\sin 85}$$

$$c = 8.25$$

$$A = \frac{1}{2} b c \sin A$$

$$A = \frac{1}{2}(9)(8.25)\sin 29$$

$$\boxed{A = 18.0 \text{ units}^2}$$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{c}{\sin 81} = \frac{10}{\sin 29}$$

$$c = \frac{10 \sin 81}{\sin 29}$$

$$c = 20.37$$

$$A = \frac{1}{2} a c \sin B$$

$$A = \frac{1}{2}(10)(20.37)\sin 70$$

$$\boxed{A = 95.7 \text{ units}^2}$$

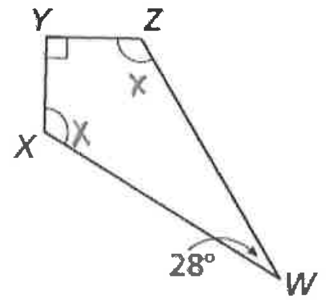
Homework:

WS 9.7A Law of Sines and Area Formula

9.7B Law of Cosines

Find the measures of ∠X and ∠Z .

1.



$28 + 2x + 90 = 360$

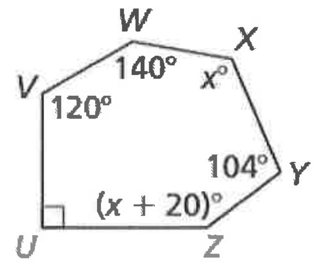
$2x + 118 = 360$

$2x = 242$

$x = 121$

$\boxed{m\angle X = m\angle Z = 121°}$

2.



$(n-2)180$

$(6-2)180$

$720°$

$90 + 120 + 140 + x + 104 + x + 20 = 720$

$2x + 474 = 720$

$2x = 246$

$x = 123$

$\boxed{m\angle X = 123°}$ $\boxed{m\angle Z = 143°}$

Theorem 9.10 Law of Cosines

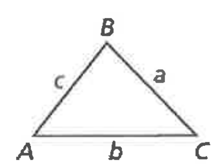
If $\triangle ABC$ has sides of length a, b, and c, as shown, then the following are true.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

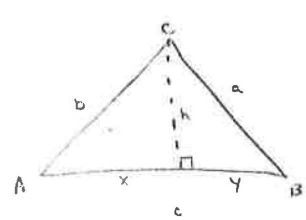
$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Proof Ex. 52, p. 516



Proof of Law of Cosines



$a^2 = h^2 + (y)^2$ $\boxed{b^2 = h^2 + x^2}$

$a^2 = h^2 + (c-x)^2$

$a^2 = (h^2 + x^2) + c^2 - 2cx$

$a^2 = b^2 + c^2 - 2cx$

$a^2 = b^2 + c^2 - 2bc\cos A.$

$\cos A = \frac{y}{b}$

$b\cos A = x$

Law of Cosines

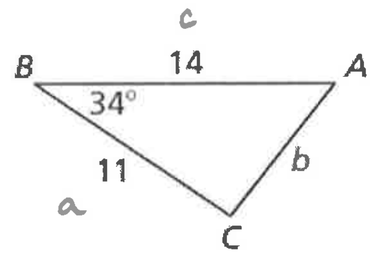
$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

* Works for SAS and SSS
* Ambiguous case for SSA

Solve the triangle. Round decimal answers to the nearest tenth.



$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 11^2 + 14^2 - 2(11)(14)\cos 34$

$b^2 = 121 + 196 - 308\cos 34$

$\sqrt{b^2} = \sqrt{317 - 308\cos 34}$

$\boxed{b = 7.9}$

$\frac{\sin B}{b} = \frac{\sin A}{a}$

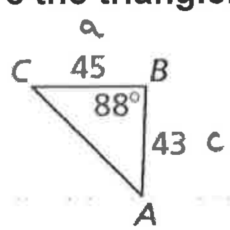
$\frac{\sin 34}{7.9} = \frac{\sin A}{11}$

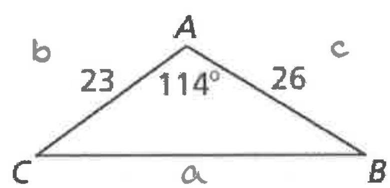
$\sin A = \frac{11\sin 34}{7.9}$

$m\angle A = 51.1°$

$\boxed{m\angle C = 94.9°}$

Solve the triangle. Round decimal answers to the nearest tenth.





$b^2 = a^2 + c^2 - 2ac\cos B$

$\sqrt{b^2} = \sqrt{45^2 + 43^2 - 2(45)(43)\cos 88}$

$\boxed{b = 61.1}$

$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin A}{45} = \frac{\sin 88}{61.1}$

$\sin A = \frac{45\sin 88}{61.1}$

$\boxed{m\angle A = 47.4°}$

$\boxed{m\angle C = 44.6°}$

$a^2 = b^2 + c^2 - 2bc\cos A$

$\sqrt{a^2} = \sqrt{23^2 + 26^2 - 2(23)(26)\cos 114}$

$\boxed{a = 41.1}$

$\frac{\sin B}{b} = \frac{\sin A}{a}$

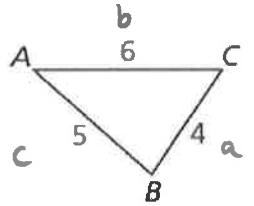
$\frac{\sin B}{23} = \frac{\sin 114}{41.1}$

$\sin B = \frac{23\sin 114}{41.1}$

$m\angle B = 30.7°$

$\boxed{\begin{array}{l} m\angle B = 30.7° \\ m\angle C = 35.3° \end{array}}$

Solve the triangle. Round decimal answers to the nearest tenth.



$a^2 = b^2 + c^2 - 2bc\cos A$

$4^2 = 6^2 + 5^2 - 2(6)(5)\cos A$

$16 = 36 + 25 - 60\cos A$

$16 = 61 - 60\cos A$

$-45 = -60\cos A$

$.75 = \cos A$

$\boxed{m\angle A = 41.4^\circ}$

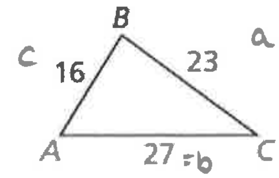
$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin 41.4}{4} = \frac{\sin B}{6}$

$\sin B = \frac{6\sin 41.4}{4}$

$\boxed{m\angle B = 82.7^\circ}$

$\boxed{m\angle C = 55.9^\circ}$



$a^2 = b^2 + c^2 - 2bc\cos A$

$23^2 = 27^2 + 16^2 - 2(27)(16)\cos A$

$529 = 729 + 256 - 864\cos A$

$529 = 985 - 864\cos A$

$-456 = -864\cos A$

$\frac{456}{864} = \cos A$

$\boxed{58.1^\circ = m\angle A}$

$\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin 58.1}{23} = \frac{\sin C}{16}$

$\sin C = \frac{16\sin 58.1}{23}$

$\boxed{m\angle C = 36.2^\circ}$

$\boxed{m\angle B = 85.7^\circ}$

Classwork:

WS 9.7B - Law of Cosines

Homework:

pg. 513 #9-12 Find the area

#13-18 Solve triangle with sines

#19-24 Solve triangle with cosines

#27-32 Solve triangle both methods

#43 Solve with Algebra